Dedication

To my parents, brothers, sisters, and to the memory of my uncle Mohamad.

Saleh Saraireh

Acknowledgment

I would like to express my sincere gratitude to my supervisor, Prof. Dr. Saod A. Alseyab, for introducing me to the exciting field of filter banks. This thesis would never have been possible without his guidance, suggestions and encouragement.

I would like to thank my colleagues for support and encouragement that help me to complete this work. I am grateful to all my friends for being good friends.

Finally, I express my sincere gratitude to my parents. They made sure that my brothers, sisters and I had the best of opportunities growing up. Their silent belief in my judgment, gave me the confidence to pursue a graduate degree. They form my support structure, and I know they will always be there for me.

Saleh Saraireh

Table of Contents

List of Figures

Abstract

Design and performance analysis of multirate digital filter banks for application in communications

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Mutah University, 2005

Multirate digital filter banks find applications in many areas of communications such as time division to frequency division multiplex translation, subband speech coding, bandwidth compression, antenna systems, and others.

Filter banks are used to carry out some type of signal processing that can not be carried out by other means. For example in time division to frequency division multiplex translation, the use of a filter can be used to increase the channel capacity. In subband coding for speech where the signal is decomposed into several bands that are coded individually, improvement in signal quality can be achieved without increasing the bit rate.

Filter banks in its most general form consists of a decomposition (or analysis) section and reconstruction (or synthesis) section.

In this thesis, design procedures for filter banks are studied and investigated and the resulting bank from each procedure is analyzed and its performance is evaluated.

These filter banks might consists of FIR or IIR channel filters. Therefore several single channel, multirate digital filters are designed first, chosen response is considered to satisfy the requirements according to specification commonly used in practical applications. Digital multirate filter banks application in transmultiplexers is given in this thesis.

ملخص

تصميم و تحليل الأداء لمجموعة المرشحات الرقمية متعددة التردد لتطبيقات الاتصالات

صالح سليمان عبدالقادر الصرايره

جامعة مؤته, 2005

مجموعة المرشحات الرقمية متعددة التردد تستخدم في مجالات متعددة و من أهمها تطبيقاتها في مجال الاتصالات, مثل تحويل (TDM) الى (FDM), ضغط المدى الترددي, تشفير الكلام, و أنظمة الهوائيات.
مجموعة المرشحات يمكن أن تنفذ معالجة لبعض الإشارات و التي لا يمكن تنفيذها بدون هذا النظام.
تتكون مجموعة المرشحات من جزئيين و هما جزء التحليل وجزء التركيب. في هذه الرسالة سوف يتم تقديم خطوات التصميم إلى مجموعة المرشحات بحيث يتم دراستها و فحصها و تحليلها و فحص أداءها.
مجموعة المرشحات إما أن تتكون من مرشحات (FIR) او (IIR), لذلك سوف يتم تصميم مرشحات رقمية ذات تردد واحد و مرشحات رقمية متعددة الترددات و مجموعة مرشحات متعددة الترددات.

Chapter One
Introduction

1.1 Motivation

Multirate digital filter banks design have become an important research topic, because its improvement can have significant impact in many engineering fields. For example, filter banks have been applied in image and video coding, speech and audio coding, communications, data transmission, and digital audio broadcasting among others. Therefore this thesis can be considered to be an attempt in this field.

1.2 Aim of Thesis

The aim of this thesis is mainly to give an insight look into the different design methods of the following:

1) Single rate digital filters.

2) Multirate digital filters.
3) Multirate digital filters banks that can be used for applications in communications, as a major application of these systems.

1.3 Introduction

Signals arise in every field of science and engineering, such as control systems, communications, radar, and others. There are two general types of signals which are continuous time and discrete time signals. In which, the continuous time signal is one that is defined at each and every instant of time, such as a voltage waveform. On the other hand the discrete time signal is one that defined at discrete instants of time, perhaps every second, minute, or day.

Filtering is a process which is used to modify, reshape, or manipulate signals according to some desired specifications. So the basic function of the filters is to remove the unwanted part of the signal. Such as elimination of the noise, and distortion brought about the signal by an imperfection transmission in the channel. Also it may be used to separate two or more distinct signals, which were mixed in order to maximize channel utilization.

The digital filter in reality is a digital system that can be used to filter discrete time signals. It can be implemented by a hardware, software (computer programs), or a combinations between the hardware and the software.

Sampling of continuous signal leads to the idea of resolution in time. The higher the sampling frequency, the better the resolution of digital approximations of the original signals. This will increase storage cost. Multirate signal processing strives to keep that cost at a reasonable level by applying smart signal processing algorithms, involving alterations of sampling rates. This process called the sampling rate conversions. So when the new sampling rate is higher than the original sampling rate, the process is called expansion, interpolation, or upsampling, since we are creating samples of the original physical process from a reduced set of sample. On the other hand, when the new sampling rate is lower than the original sampling rate, the process is called decimation, or downsampling.

Multirate signal processing consists of using different sample rates within a system to achieve computational efficiencies that are impossible to obtain with a system that operates on a single fixed sample rate.

Multirate digital filter banks are fundamental systems commonly used in digital signal processing (DSP). Typically, they are used to decompose a discrete-time signal into a set of frequency selective components called subband signals. Filter banks are collection of filters, with a common input or output signal.

Filter banks consist of four sections, which are the analysis filters, synthesis filters, decimation factor, and the interpolation factor.

Multirate digital filter banks can be divided based on the way the signal is blocked and sent though the channels: uniform when all the channels are down-sampled by the same factor and non-uniform otherwise. Polyphase implementation is one of the great methods that can be used to design a multirate digital filter banks.

Multirate digital filter banks have important applications, such as transmultiplexing, which converts signals from TDM (time division multiplexing) into FDM (frequency division multiplexing), speech processing, audio/video signal compression, signal detection and spectral analysis and others.

1.4 Scope of Thesis

This thesis comes as an attempt to design FIR and IIR digital filters for single rate, multirate, and multirate digital filter banks.

The design of a single rate FIR and IIR digital filters is presented in chapter two. The method used to design an FIR digital filter is a Kaiser window since it is a parametric method which has shapes that are determined by different parameters (which are the filter length (N+1) and the parameter (β)), and can be adjusted to satisfy given constraints on the frequency response. Elliptic IIR digital filter is presented in this chapter too, since it presents equiripple in both passband and stopband, which yield a lower filter order.

A study of the digital filters types, which involves, filter design specifications, type of digital filters, FIR digital filters design, window design methods, Kaiser window, some examples of FIR digital filters design, IIR filter design, the bilinear transformation, analog filters, elliptic filters design, some examples of IIR digital filters design, and comparison between FIR filters and IIR filters is included in chapter two.

Multirate filters and systems are presented in chapter three, where the multirate systems are used when more than one sampling rate is used. The process of increasing the sampling rate by some integer factor is done by the interpolator, while the process of decreasing the sampling rate is done by the decimator. The design of FIR and IIR multirate digital filters in this chapter comes as a continuation of FIR and IIR digital filter design in chapter two, so the same methods are used. The filter order of the FIR and IIR interpolator is the order of the original FIR or IIR filter times the interpolator factor. On the other hand, The filter order of the FIR and IIR decimator is the order of the original FIR or IIR filter divided by the decimator factor. Also the passband

edge frequency and the stopband edge frequency of the interpolator filter are the passband or the stopband edge frequency of the original filter divided by the interpolation factor, while the passband edge frequency and the stopband edge frequency of the decimator filter are the passband or the stopband edge frequency of the original filter times the decimation factor.

Filter banks are collection of filters, with a common input or output signal, these filters operate in a parallel structure mode, either in an analysis mode with a single input and multiple outputs or in a synthesis mode with multiple inputs and a single output. The filter banks that will be designed in this thesis is a uniform multirate digital filter banks, where each filter in this system has the same bandwidth. The uniform multirate digital filter bank will be designed here is the uniform DFT filter bank. Uniform DFT filter banks may be efficiently implemented using the polyphase implementation employing fast transforms such as the discrete Fourier transform (DFT), and the noble identity properties of decimation and interpolation. FIR and IIR uniform DFT multirate digital filter banks will be designed in this thesis that can be used for applications in communications such as a transmultiplexer, subband speech coding, antenna systems and others.

Chapter three comes to show the fundamentals of multirate signal processing system that can be used in multirate filter banks design. This chapter discusses discrete time signal processing, fundamental building blocks, sampling rate conversion, decimation/interpolation filter systems, noble identities, polyphase representation, multirate digital filters, FIR decimator and interpolator, IIR decimator and interpolator, and some examples design of FIR and IIR interpolator and decimator are given.

Chapter four contains the design procedures of multirate digital filter banks which can be used in many applications, by using the polyphase implementation. So this chapter gives a basic study of perfect reconstruction, polyphase representation, multirate digital filter bank design, uniform digital filter bank, uniform DFT filter banks, design procedures of multirate digital filter banks, analysis filter bank design, synthesis filter bank design, the overall system (Analysis and Synthesis) filter banks design, and some examples design of FIR and IIR uniform DFT multirate digital filter banks

The applications of multirate digital filter banks are discussed in chapter five, which comes as a major applications in multirate digital filter banks in communications.

Chapter Two
Types of Digital Filters and Their Design Methods

2.1 Introduction

A key element in processing of signals is the filter. In signal processing, the function of a filter is to remove unwanted parts of the signal, such as random noise, or to extract useful components lying within a certain frequency range called spectrum.

The function of digital filter is the same as its analog filter counterpart, but its implementation is very different. Analog filters are implemented using either active or passive electronic circuits, and they operate on continuous waveforms. On the other hand, digital filters are implemented using either a digital logic circuit or computer programs and they operate on a sequence of numbers that are obtained by sampling the continuous waveform. Today there is widespread of digital filters, because of easy availability of computers [1]. A computer program can be written to implement any type of digital filters.

Digital filter has several advantages over analog filter. A digital filter is highly immune to noise because of the way it is implemented. Accuracy is dependent only on round- off error, which is directly determined by the number of bits that the designer chooses for representing the variable in the filter. Also it is easy and inexpensive to change filter operating characteristics.

There are two basic types of digital filters, non-recursive and recursive filters[1]. For non-recursive filters, the transfer function contains a finite number of elements. but for recursive filters, the transfer function is expressed as the ratio of two polynomials.

This chapter is mainly concerned with the study of basic digital filters, which particularly includes, filter design specifications for types of digital filters, FIR digital filters design, particulary window design methods, and IIR filter design, especially the bilinear transformation method from analog filter prototype satisfying an elliptic response. Comparison between FIR filters and IIR filters is given briefly.

2.2 Filter Design Specifications.

When talking about filter design the required specifications of the filter must be satisfied, the specifications that must be considered are:

1) The passband ripple δ_p.
2) The stopband ripple δ_s.
3) The stopband attenuation E_s in dB which is equal to

$$E_s = -20\log_{10} \delta_s \tag{2.1}$$

4) The passband ripple attenuation E_p in dB which equal to

$$E_p = -20log_{10}(1- \delta_p) \tag{2.2}$$

5) The passband edge w_p.
6) The stopband edge w_s.
7) The transition bandwidth in radians Δw which is equal to

$$\Delta w = w_s - w_p \tag{2.3}$$

This specification for the filter shown in Fig. (1).

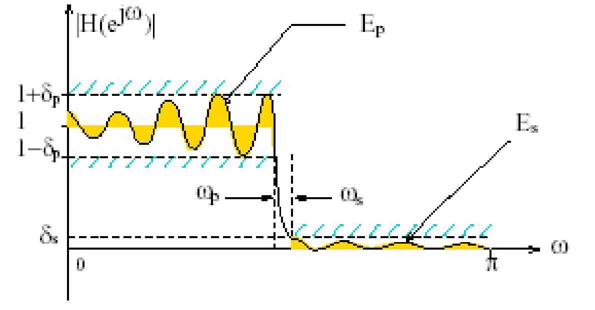


Figure (1)
Magnitude response specifications for lowpass filters.

The variable $f=w/2\pi$ is said to be the normalized frequency [2]. Frequency response plots in the range $0 \leq w \leq \pi$ correspond to the range $0 \leq f \leq 0.5$ in term of f.

By using these specifications we can find the magnitude response of the filter, the phase response of the filter, and the order of the filters can also be determined.

It is obvious that the larger the order of the filter, the more computation is required. It will also be more costly if it is implemented with dedicated hardware [3], so we like to design a filter with an order as small as possible to satisfy the given specifications.

The design of digital filter is a mature field, in fact, FIR and IIR filter design is greatly facilitated by the availability of numerous computer software programs such as Matlab.

2.3 Types of Digital Filters

The transfer function $H(z)$ of a digital filter is usually represented in the z –transform domain, which equals to the z-transform of the output sequence $Y(z)$ divided by the z- transform of the input sequence $X(z)$, which is given by

$$H(z)=Y(z)/X(z) \tag{2.4}$$

There are generally two basic types of the digital filters, which are non-recursive and recursive filters. The transfer function for non-recursive filter has the following form of a polynomial

$$H(z)=\Sigma_i h_i z^{-i}=h_0+h_1 z^{-1}+\dots\dots\dots+h_n z^{-n} \tag{2.5}$$

where $i=0,1,\dots\dots,n$, and the constant value $h_0, h_1, \dots, h_n$ are the filter coefficients.

While the transfer function of the recursive filter has the following form

$$H(z)= \Sigma_i a_i z^{-i}/(1-\Sigma_i b_i z^{-i})$$

$$= a_0+a_1 z^{-1}+\ldots\ldots\ldots+a_n z^{-n}/1-b_1 z^{-1}-\ldots\ldots-b_n z^{-n} \qquad (2.6)$$

where i=0,1,………, n and n is the order of the filter.

The value of z which make $H(z)$ equal to zero are called the zeros of the transfer function, while the value of z which make $H(z)$ equal to infinity are called the poles.

The non-recursive filter associated with digital filters whose impulse response is finite in duration (i.e., the impulse response of the digital filters is zero outside a finite interval) so it is denoted as finite impulse response (FIR) filters. While the recursive filters are associated with impulse responses whose duration are infinite and they are referred to as infinite impulse response (IIR) digital filters [4].

FIR filters are generally realized non-recursively, which means that there is no feedback involved in computation of the output data. So the output data depends only on the present and past inputs. While IIR filters can be realized recursively, which means that there is a feedback involved in computation of the output data. So the output data depends on the input and the output data.

2.4 FIR Digital Filters Design

A digital filter is called a finite impulse response (FIR) if its impulse response has a finite number of nonzero entries such as $h[n]$, for n=0,1,……,N. Generally we assume implicitly $h[0]\neq 0$ and $h[N]\neq 0$. The filter has at most N+1 nonzero entries and is said to have length N+1 [3].

FIR filters are a non-recursive filters, which can be characterized by a transfer function $H(z)$ given by

$$H(z)=\sum_{i=0}^{N} h_i z^{-i} \qquad (2.7)$$

FIR filters can be designed to have linear phase characteristic, consequently, its impulse response satisfies the condition

$$h(n)=\pm h(N-1-n) \qquad n=0, 1,\ldots\ldots, N-1 \tag{2.8}$$

The linearity of the phase response means that different frequencies will have the same delay as they pass through the filter. In practice, FIR filters can be used, when there is a requirement for a linear phase characteristic within the passband of the filters [5].

The magnitude and phase response of a particular filter is computed from the transfer function of this filter, as we know that the transfer function of the digital filter is represented by z-domain where z equal to (e^{jw}), so to compute the magnitude and phase response we use the frequency domain (e^{jw}) instead of the z-domain, then we can find the magnitude and phase function for this filter and plot it for particular values of frequency. This will be applied in all examples in this thesis.

When all the zeros of the FIR filter are inside the unit circle, then the FIR system is called the minimum phase system, on the other hand when all the zeros of the FIR filter are outside the unit circle, then the FIR system is called the maximum phase system.

FIR system with some zeros inside the unit circle and the others are outside the unit circle, this system is called mixed phase system.

The ideal lowpass FIR filter $H_i(e^{jw})$ with cutoff frequency (w_c) is given by

$$H_i(e^{jw}) = \begin{cases} 1, & -w_c<w<w_c \\ 0, & \text{otherwise} \end{cases} \tag{2.9}$$

This filter is zero-phase, with magnitude equal to unity in the passband and zero in the stopband. The impulse response of the ideal lowpass FIR filter (the inverse transform of $H_i(e^{jw})$ is given by

$$h_i(n) = \frac{w_c}{\pi}\left(\frac{\sin(w_c n)}{w_c n}\right) \qquad -\infty<n<\infty \tag{2.10}$$

The impulse response of the ideal lowpass filter does not satisfy the BIBO (Bounded Input Bounded Output) stability requirement. So the ideal filter is not stable. To obtain an FIR filter from this, the impulse response must be truncated to a specific interval, so

$$h(n) = \begin{cases} h_i(n), & -N/2\le n\le N/2 \\ 0, & \end{cases}$$

(2.11)

otherwise

This truncation causes a ripples in the passband and stopband , and the ripple size grows as we get closer to the cutoff frequency (w_c). As N increase, the ripple get crowded closer to the cutoff frequency (w_c), but the size of the peak ripple does not decrease. This is called the *Gibbs phenomenon* [2].

The most commonly used method that can be used to design the digital FIR filters is the windowing method [6], which is based on multiplication the filter function by another function known as the window function.

2.5 Window Design Method

The window method comes as improvement over the truncation technique. The window method is widely used, and in principle it is based on multiplying the filter function by some specially chosen window function.

This method begins with the desired frequency response specification $H_i(e^{jw})$ and determine the corresponding unit sample response $h_i[n]$. In general, the unit sample response $h_i[n]$ obtained is infinite in duration and must be truncated at some point, say at n=N-1, to yield an FIR filter of length N . Truncation of unit impulse response $h_i[n]$ to a length N-1 is equivalent to multiplying $h_i[n]$ by a rectangular window [7].

Thus the unit sample response of the FIR filter becomes

$$h[n] = h_i[n]v(n) \tag{2.12}$$

where $v(n)$ is a window function, which is zero for $|n| > N/2$.

Two parameters of the window which control the quality of the filter response are:

(a) The main lobe width ΔB of the window function, which controls the filter transition bandwidth Δw.

(b) The peak sidelobe level of the window function, which controls the peak passband and stopband ripples of the response [2].

The most commonly used window functions are:

1. Rectangular window.
2. Hamming window.
3. Blackman window.
4. Kaiser window.

The truncation of the impulse response is done using both non-parametric and parametric windows. Non-parametric windows have a fixed

shape and include Rectangular, Blackman, and Hamming window functions. The use of these windows imposed a fundamental tradeoff between main lobe width and side lobe height. On the other hand parametric window functions, such as the Kaiser window, have shapes that are determined by different parameters (which are the filter length (N+1) and the parameter (β)), and can be adjusted to satisfy given constraints on the frequency response. So that Kaiser widow will be used in the design of FIR filters in this thesis, since it can be adjusted to satisfy given constraints on the frequency response.
The rectangular window is given by

$$v(n) = \begin{cases} 1, & -N/2 \le n \le N/2, \\ 0, & \text{otherwise} \end{cases} \tag{2.13}$$

The Hamming window is given by

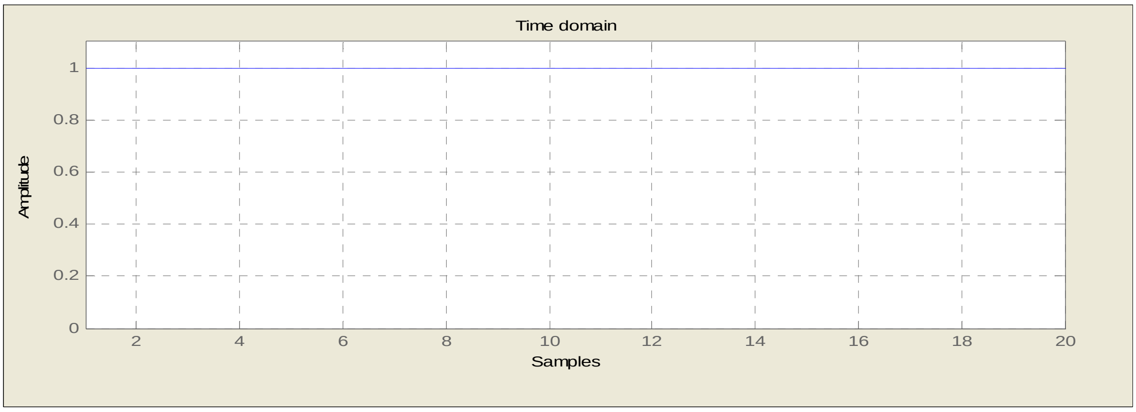
$$v(n) = \begin{cases} 0.54 + 0.46\cos(2\pi n / N), & -N/2 \le n \le N/2, \\ 0, & \text{otherwise} \end{cases} \tag{2.14}$$

The Blackman window is given by

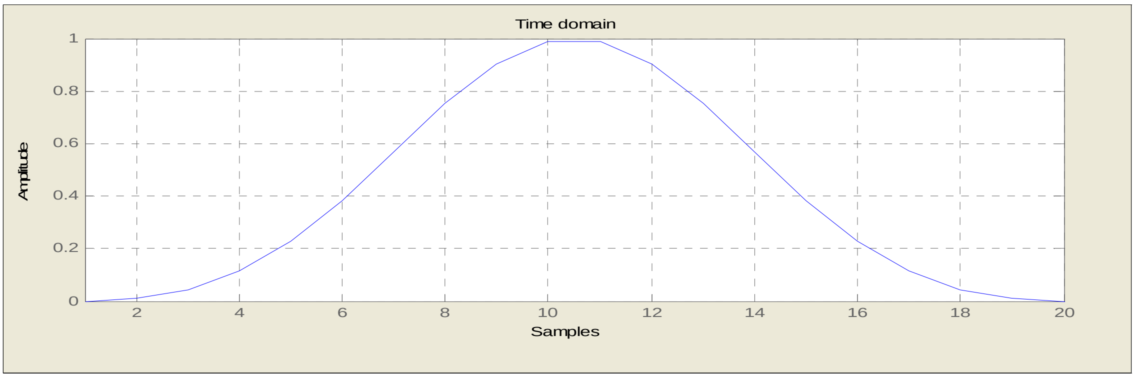
$$v(n) = \begin{cases} 0.42 + 0.46\cos(2\pi n / N) + 0.08\cos(4\pi n / N), & -N/2 \le n \le N/2, \\ 0, & \text{otherwise} \end{cases} \tag{2.15}$$

We see that all windows equal to zero for $n<-N/2$ and $n>N/2$, and have the symmetric property. Their envelopes are plotted in Fig. (2) for N=20.

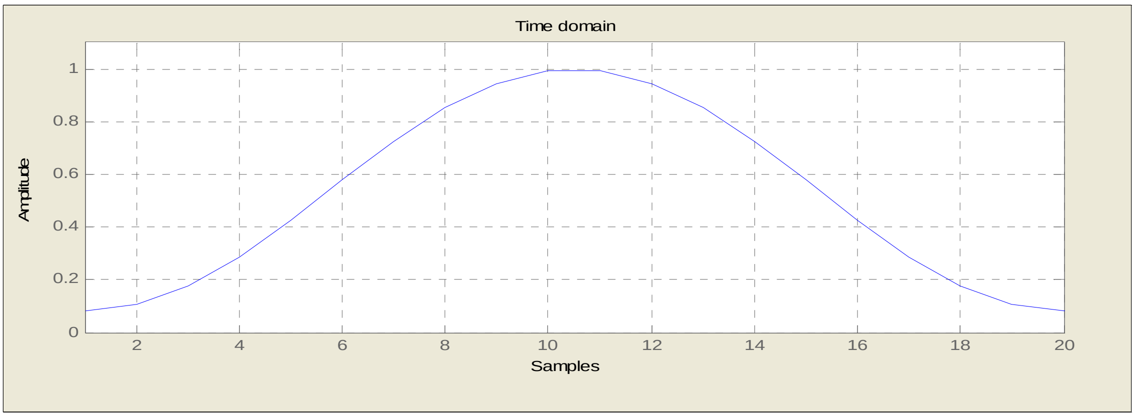
The direct truncation of the response is the same as multiplying the response by the rectangular window in eq. (2.13), the spectra of rectangular window has one main lobe and a number of side lobes, the leakage in the neighborhood of w_c is due to the main lobe, and the ripples is due to the side lobes. Hamming window can reduce the side lobes as compared with the rectangular window, the additional cosine term in Blackman window function leads to a further reduction of the amplitude of the *Gibbs* oscillations.


Time domain
Amplitude
Samples
1
0.8
0.6
0.4
0.2
0
2
4
6
8
10
12
14
16
18
20

(a)


Time domain
Amplitude
Samples
1
0.8
0.6
0.4
0.2
0
2
4
6
8
10
12
14
16
18
20

(b)



(c)

Figure (2)
The envelope of the windows (a) rectangular (b) Blackman (c) Hamming.

2.5.1 Kaiser window:

The Kaiser window is also known as parametric window functions, because the shapes of the magnitude response of the filter is determined by different parameters filter length and the parameter (*ß*), that can be adjusted to satisfy given constraints on the frequency response. The value of (*ß*) is determined by the stopband attenuation requirements.

Kaiser window is actually a family of windows spanned by a parameter beta (*ß*). By adjusting *ß*, one obtains any desired stopband attenuation for the response. The order of the filter is adjusted to satisfy the requirement on the transition width [2].

The Kaiser window is given by:

$$v(n) = \begin{cases} I_0[\beta\sqrt{1-(n/0.5N)^2}\,]/I_0(\beta), & -N/2 \le n \le N/2, \\ 0, & \text{otherwise} \end{cases} \tag{2.16}$$

where $I_0(x)$ is the modified zeorth-order Bessel function, which can be computed from the power series [2],

$$I_0(x) = 1 + \sum_{k=1}^{\infty}\left[\frac{(0.5x)^k}{k!}\right]^2 \tag{2.17}$$

note that when $v(n)$ is computed in this manner, the coefficients of the lowpass filter can be found by using eq. (2.7).

The parameter β, which depends on the attenuation requirement can be found by

$$\beta = \begin{cases} 0.1102(E_s - 8.7), & \text{If } E_s > 50 \text{ dB.} \\ 0.5842(E_s - 21)^{0.4} + 0.07886(E_s - 21), & \text{If } 21 < E_s < 50 \text{ dB.} \\ 0 & \text{If } E_s < 21 \text{ dB.} \end{cases} \quad (2.18)$$

Note that the parameter β depends on the stopband attenuation of the filter. Now the filter order (N) is given by

$$N = (E_s - 7.95)/\ 14.36(\Delta f) \quad (2.19)$$

Design example (2.1)

Required to design an FIR lowpass digital filter by using Kaiser window which satisfying the following specifications:
The passband edge frequency (w_p)=0.2π, the stopband edge frequency (w_s)=0.3 π, the minimum stopband attenuation (E_s)=40dB, and the passband ripple (δ_p) = the stopband ripple (δ_s).

Solution:

The cutoff frequency (w_c)=(w_p+w_s)/2 = 0.25 π.

E_s=-20log$_{10}$ δ_s =40dB.

$\delta_s = (10)^{-2} = 0.01$.

since $\delta_p = \delta_s$, then the value of $\delta 1$=0.01 so the peak passband attenuation (E_p) is:
$E_p = -20 log_{10}(1 - \delta_p) = -20 log_{10}(1-0.01) = 0.087$dB.

By using eq.(2.17) and eq. (2.18) we find that the value of parameter (β) equal to 3.395. The transition bandwidth (Δf) equal to

$\Delta f = (w_s - w_p)/2\pi = (0.3\,\pi - 0.2\,\pi)/2\,\pi = 0.05$

Now by using eq. (2.19) we find that the filter order (*N*) equal to 45.

The magnitude response of the FIR lowpass filter which meets the required specifications is shown in Fig. (3). From Fig. (4) we see that the lowpass FIR filter has a linear phase response which means that, different frequencies will have the same delay as they pass through this filter.

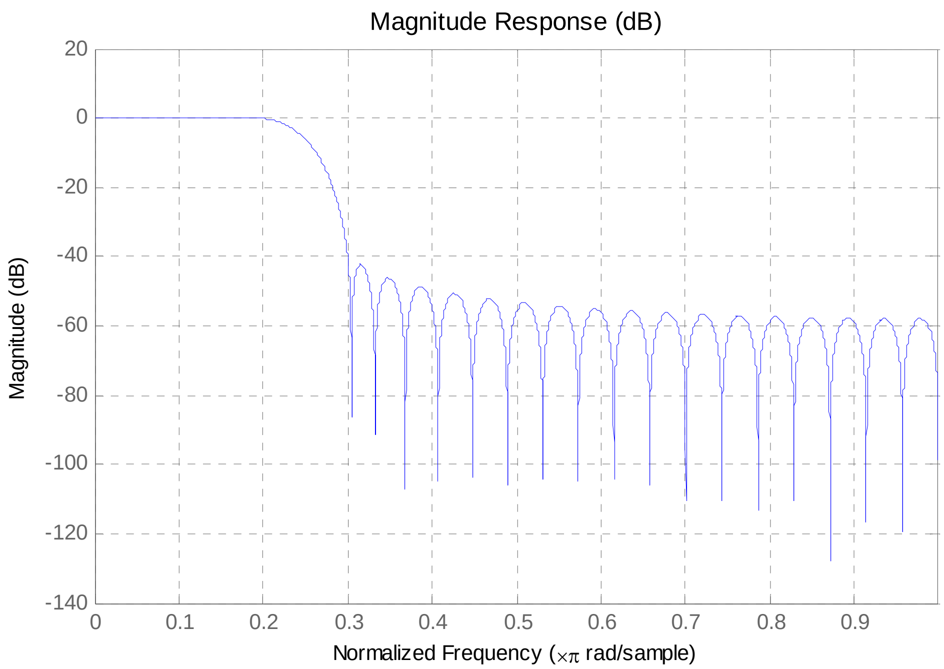


Figure (3)
Magnitude response of the FIR lowpass filter with filter order 45.

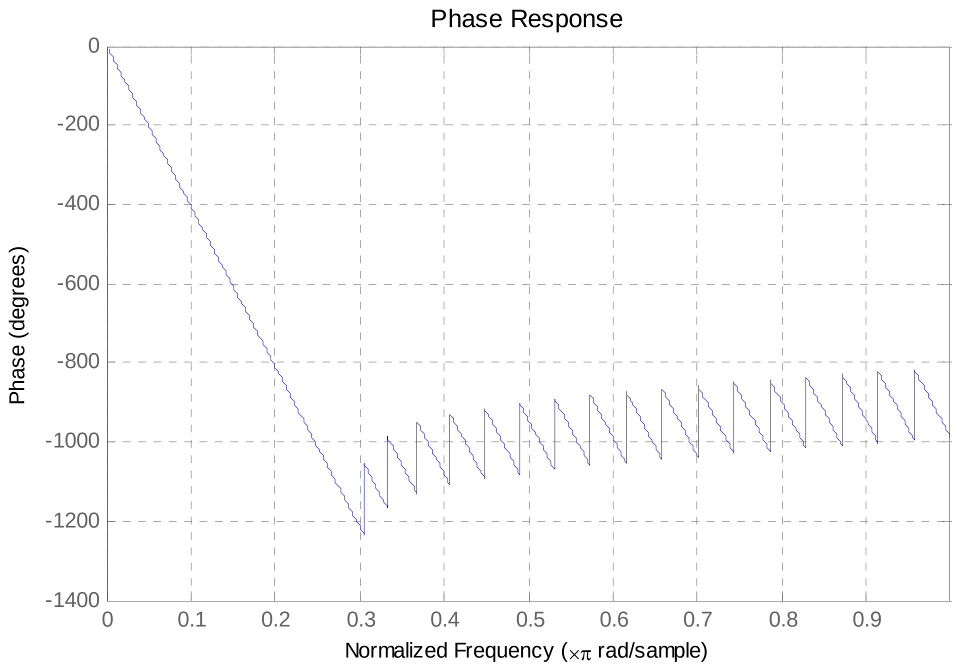


Figure (4)
Phase response of the FIR lowpass filter with filter order 45.

Now if the value of peak ripple decreases, the filter order (N) will increase, so as the filter order (N) increase, the ripples get crowded and closer to the cutoff frequency (w_c) as we will see in the following example.

Design Example (2.2)

Suppose that the same specifications is used as in the previous example, except the ripple $\delta_p = \delta_s = 0.001$. Design a lowpass FIR digital filter using the Kaiser window method.

Solution:

The cutoff frequency (w_c) is the same, also the transition bandwidth (Δf) is the same, while the stopband attenuation (E_s) will change to

$E_s = -20\log_{10} \delta_s = -20\log_{10} (0.001) = 60\text{dB}$

Also the peak passband attenuation (E_p) is:

$E_p = -20 log_{10}(1 - \delta_p) = -20 log_{10}(1 - 0.001) = 0.0087\text{dB}.$

By using eq.(2.10) and eq. (2.11) we find that the value of parameter (*β*) equal to 5.6533. And by using eq. (2.12) we find that the filter order (*N*) equal to 73.

The magnitude response is shown in Fig. (5), and the phase response is shown in Fig. (6). We can see that the lowpass FIR digital filter has a linear phase response, so that if different frequencies passing through this filter, they will have the same delay.

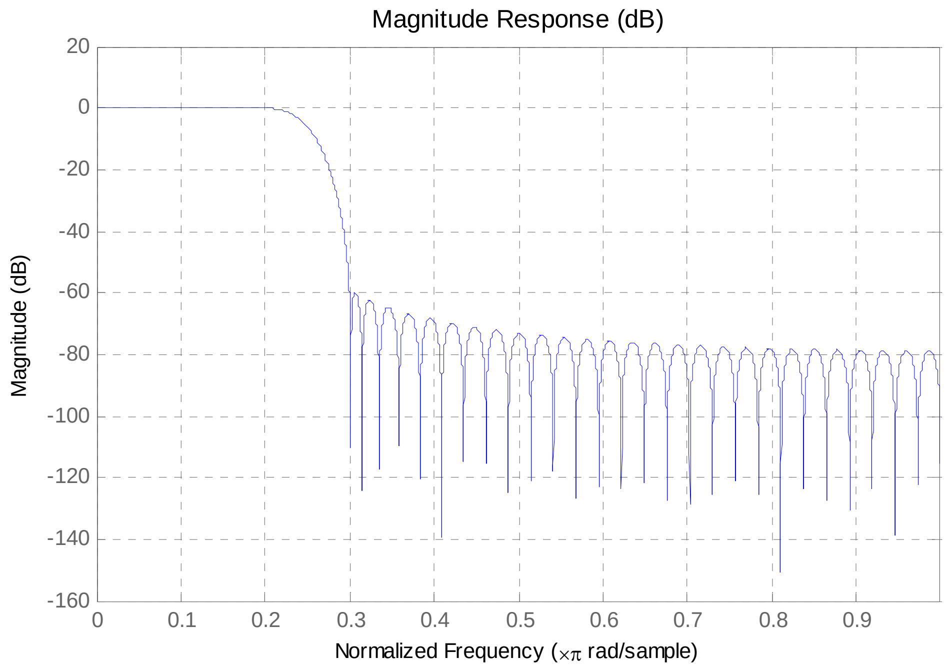


Figure (5)
Magnitude response of the lowpass FIR filter with filter order 73.

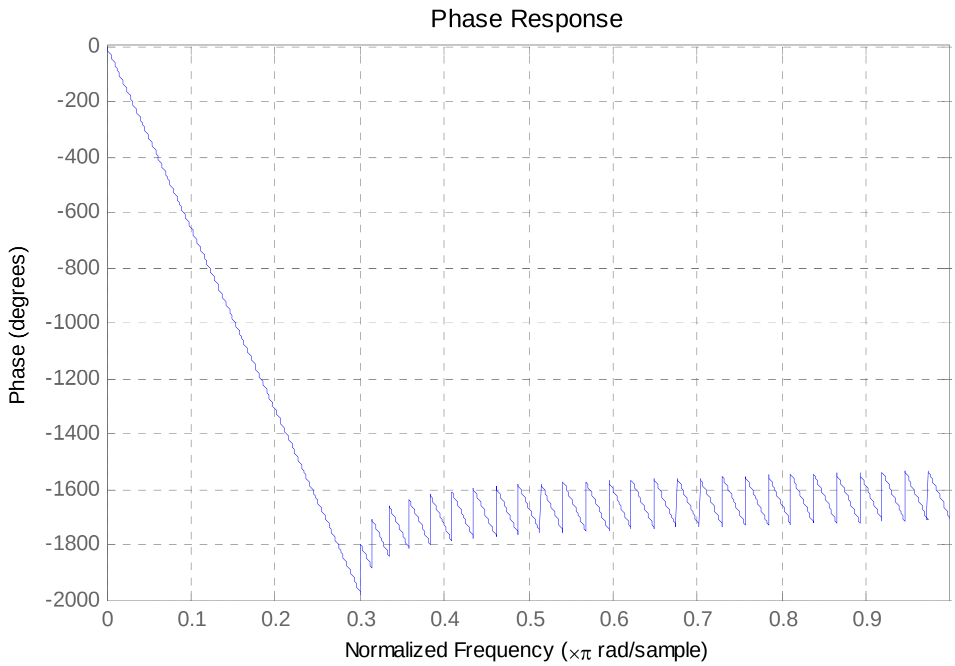


Figure (6)
Phase response of the lowpass FIR filter with filter order 73.

2.6 IIR Filter Design

The general format of the transfer function of IIR filters is expressed as the ratio of two polynomials of the form

$$H(z) = P(z)/D(z) \tag{2.20}$$

Where $P(z)$ and $D(z)$ are polynomials in z^{-1}. The zeroes of $p(z)$ located on the unit circle, so it has the form $e^{(jw)}$.

The output from an IIR digital filter is made up from previous inputs and previous outputs, as described by the difference equation

$$y[n] = -\sum_{k=1}^{N} a_k x[n-k] + \sum_{k=0}^{M} b_k y(n-k) \tag{2.21}$$

In order to compute the impulse response two convolutions are involved, one with the previous inputs, and one with the previous outputs. In each case the convolving function is called the filter coefficients.

If such a filter is subjected to an impulse then its output need not necessarily become zero after the impulse has run through the summation. So

the impulse response of such a filter can be infinite in duration. Such a filter is called an Infinite Impulse Response filter or IIR filter.

Note that eq. (2.21) describes a linear time-invariant system, which is causal and physically realizable. It is easy to demonstrate that its frequency response is

$$H(w) = \frac{\sum_{k=0}^{M} b_k e^{-jwk}}{1 + \sum_{k=1}^{N} a_k e^{-jwk}} \tag{2.22}$$

It is expected that an IIR filter phase response is non-linear. The non-linearity of the phase response means that different frequencies will have different delay as they pass through the filter. The lose of phase linearity is attributed to the lack of symmetry of the impulse response. In practice, IIR filters can be used, when there is no requirement for linear phase characteristic.

The methods that can be used to design IIR filters differ from the methods that can be used to design FIR filters, since the transfer function of IIR filter is a ratio of polynomials of z, while the transfer function of FIR filter is a polynomial of z^{-1}.

There are several methods can be used to design an IIR filters such as

a) Invariant impulse response method.
b) Matched-z-Transformation method.
c) Bilinear transformation method.

Invariant impulse response method is the simplest approach, requiring to design a prototype polynomial to be scaled to the desired cutoff frequency, and then solved for the required values. The aliased components due to the sampling, make this filter impractical for many applications[8].

Matched-z-Transformation method is a special case of the impulse invariant method, which maps both zeros and poles directly from the s-domain into the z-domain. It is unstable, but it processes no particular virtues in term of its time or frequency domain properties and hence is not widely applied. Superior closed form digital filter designs use the bilinear transformation as this avoids any deleterious effects. It is the most frequently used design method and is described further bellow[8].

2.7 The Bilinear Transformation

This technique describes a mapping from the s-plane to the z-plane, so it is called the bilinear transformation.

The transfer function in Laplace transform is converted to a transfer function in z-transform by the following equation

$$s= 1-z^{-1}/1+z^{-1} \tag{2.23}$$

The result is a rational function $H(z)$ in the variable z^{-1}.
With s=jΩ i.e on the imaginary axis of the s-plane, z will be equal to e^{jw}, then w will be as follow

$$\Omega=tan(w/2) \tag{2.24}$$

Which shows that at Ω=0, then w=0, and at $w=\pi$ then $\Omega=\infty$.

The band edges w_p and w_s corresponding to the analog filter response are determined by

$$w_p=2arctan(\Omega_p) \tag{2.25}$$

and

$$w_s=2arctan(\Omega_s) \tag{2.26}$$

The size of the ripples $\delta 1$ and $\delta 2$ are unchanged.

To design a digital filter by linear transformation method with digital filter specifications w_p, w_s, $\delta 1$, and $\delta 2$. The following steps must be followed:

a) Find Ω_p which is equal to

$$\Omega_p=tan(w_p/2) \tag{2.27}$$

and Ω_s which equal to

$$\Omega_s=tan(w_s/2) \tag{2.28}$$

b) Design the corresponding analog filter for the specification Ω_p, Ω_s, $\delta 1$, and $\delta 2$.
c) Transform the transfer function from Laplace transform to z-transform by using the bilinear transformation in eq. (2.23).

2.8 Analog Filter

The magnitude square response of the analog filters take the form

$$|\ H_a(j\Omega)\ |^2 = \frac{1}{1+F^2(\Omega)} \tag{2.29}$$

where $F(\Omega)$ is a real-valued rational function of Ω. Note that $|\ H(j\Omega)\ | \leq 1$.

There are several types of lowpass analog filters responses, the most commonly used as the well known responses such as

1- Butterworth (or Maximally Flat) filter.
2- Type I Chebychev filter.
3- Type II Chebychev filter.
4- Elliptic filter.

Each one of these filters has special properties and specifications, also some of these filters come as an improvement of another filter.

Low pass Butterworth filters are all-poles filters characterized by having a frequency response which is monotonic in both the passband and stopband, Butterworth filters are fully specified by the cutoff frequency w_c, and the filter order N.

The magnitude square response of the Butterworth filter is given by

$$|H_a(j\Omega)|^2 = \frac{1}{1+(\Omega/\Omega_c)^{2N}} \tag{2.30}$$

Note that the magnitude response of Butterworth filter varies from unity at Ω=0, to zero at $\Omega=\infty$. Ω_c is called the 3 dB point, since the magnitude response of the lowpass Butterworth filter at cutoff frequency Ω_c corresponds to 3 dB. The Butterworth filter is also called maximally flat, since the first derivative of $|H_a(j\Omega)|^2$ at Ω=0 equal to zero. Note that the Butterworth filter is monotonically in both passband and stopband.

The Chebyshev filter response is a mathematical strategy for achieving a faster roll off by allowing ripples in the frequency response.

The magnitude response of the Chebyshev I filter is given by

$$|H_a(j\Omega)| = \frac{1}{\sqrt{1+\varepsilon^2 V_N^2(\Omega/\Omega_p)}} \tag{2.31}$$

where $V_N(v)$ is the Nth order Chebyshev polynomial and given by

$$V_N(v)=2vV_{N-1}(v) - V_{N-2}(v) \qquad \text{for } N>2 \tag{2.32}$$

$$V_1(v)=v \tag{2.34}$$

$$V_2(v)=2v^2 - 1 \tag{2.35}$$

and ε is a parameter of the filter related to the ripple in the passband

$$\varepsilon^2 = 10^{0.1(E_p)} - 1 \tag{2.36}$$

The polynomial $V_N(v)$ is an even function of v if N is even and an odd function if N is odd. Its square, however, is always even. The polynomials have the property $V_N(1)$ for all N. Its value at v=0 is 0 for N odd, and 1 or -1 for N even. The polynomial oscillates between 1 and -1 for $-1<v<1$ and grows monotonically to $\pm\infty$ as $v \rightarrow \infty$. Using this property, Chebyshev I filter will be equiripple in the passband and monotonic in the stopband [3].

Chebyshev II filter can be developed from Chebyshev I filter, while Chebyshev I filter is equiripple in passband and monotonic in stopband, Chebyshev II filter is equiripple in stopband and monotonic in passband, so Chebyshev II filter also called Inverse Chebyshev.

The magnitude response of the Chebyshev I filter is given by

$$\left|H_a(j\Omega)\right| = \frac{\varepsilon\ V_N(\Omega_p/\Omega\ )}{\sqrt{1+\varepsilon^2 V_N^2(\Omega_p/\Omega_s)}} \tag{2.37}$$

Note that Chebyshev I filters are all poles filters that exhibit equiripple behavior in the passband and a monotonic behavior in the stopband. On the other hand, Chebyshev II filters contain both poles and zeros and exhibit a monotonic behavior in the passband and an equiripple in the stopband.

Chebyshev I filter is much better passband characteristics and Chebyshev II filter is much better stopband characteristics than Butterworth filter. An improved passband and stopband characteristics can be achieved by using elliptic approximation in which the passband attenuation oscillates between zero and maximum passband attenuation E_p and the stopband attenuation oscillates between infinity and a prescribed minimum stopband attenuation E_s [6]. Because of the equiripple nature in both passband and

stopband, the elliptic filter requires a smaller order than other filters for the same specifications [2]. Hence the design of elliptic filter is adapted in this thesis as the design prototype.

The elliptic filter has a magnitude response that is equiripple both in the passband and stopband. The magnitude response for the elliptic filter is given by

$$|H_a(j\Omega)| = \frac{1}{\sqrt{1+\varepsilon^2 U_N^2(\Omega \;/\Omega_p)}} \tag{2.38}$$

where $U_N(\Omega)$ is function that has the following property: $U_N(1/\Omega)= 1/U_N(\Omega)$. The roots of the numerator lie in the interval $0<\Omega<1$ and the roots of the denominator lie in $1< \Omega <\infty$.

2.9 Elliptic Filters Design

The elliptic filter is an improved version of the Chebyshev filter, in which the response function is chosen to be of the form similar to the well known elliptic function. This filter distributes the passband ripple over the passband and the stopband.

The elliptic filter belongs to the class of Infinite Impulse Response (IIR) filters, because its response to an impulse input remains non-zero until infinite time.

An elliptic filter comes as an improvement over Chebyshev filters, where the elliptic filter exhibits equiripple behavior in both passband and stopband[2].

The filter order is given by

$$N \geq (\log(16D)/\log(1/q)) \tag{2.39}$$

where

$$D = 10^{0.1E}{}_s - 1 \;/\; 10^{0.1E}{}_p - 1 \tag{2.40}$$

$$q_0 = \frac{1}{2}\left(\frac{1-\sqrt{k'}}{1+\sqrt{k'}}\right) \tag{2.41}$$

The parameter q is called the modular constant, which is given by:

$$q = q_0 + 2q_0^{5} + 15q_0^{9} + 150q_0^{13} \quad (2.42)$$

$$k' = \sqrt{1-k} \quad (2.43)$$

$$k = k_0 \quad (2.44)$$

$$k_0 = \tan(\Omega_s/2)/\tan(\Omega_p/2) \quad (2.45)$$

When we design an elliptic filter, the frequency normalization condition must be satisfied, which means that the band edges are related as

$$\Omega_p\ \Omega_s = 1 \quad (2.46)$$

if this condition is not satisfied then define $\Omega_p^{`}$ and $\Omega_s^{`}$ as follows

$$\Omega_p^{`} = \alpha\ \Omega_p \quad (2.47)$$

and

$$\Omega_s^{`} = \alpha\ \Omega_s \quad (2.48)$$

where

$$\alpha = (\Omega_p \Omega_s)^{-0.5} > 0. \quad (2.49)$$

This will ensure that the frequency normalization condition $\Omega_p^{`}\ \Omega_s^{`} = 1$ holds.

In this case, an analog elliptic filter $H_a^{`}(s)$ can be designed whose specification are $\Omega_p^{`}$, $\Omega_s^{`}$, $E_{max}$, and $E_s$. By replacing $H_a(s)$ by $H_a^{`}(\alpha s)$, then $H_a(s)$ meets the specifications Ω_p, Ω_s, E_{max}, and E_s. Now by applying bilinear transformation, we obtain the digital filter.

$$H(z) = H_a(s)|_{s=(1-z^{-1})/(1+z^{-1})} \quad (2.50)$$

Design Example(2.3)

Required to design a digital lowpass IIR elliptic filter with the following specifications: The passband edge frequency (w_p)=0.15л, the stopband edge

frequency (w_s)=0.2 л, the passband ripple (δ_p)=0.01, and the stopband ripple (δ_s)=0.001.

Solution;

The passband ripple attenuation (E_p) = -20log(1- δ_p) = 0.087dB.

The minimum stopband attenuation (E_s) = -20log(δ_s) = 60dB.

$\Omega_p = tan(w_p/2) = tan(0.15 \text{л}/2) = 0.24$

$\Omega_s = tan(w_s/2) = tan(0.2 \text{л}/2) = 0.325$

To design an elliptic filter the normalization condition must be satisfied

$\Omega_p \Omega_s = (0.24*0.325)=0.078 \neq 1$, so

$\alpha=(\Omega_p \Omega_s)^{-0.5} = 3.58$

$\Omega_p^{`} = \alpha \, \Omega_p = 0.86$

$\Omega_p^{`} = \alpha \, \Omega_p = 1.1635$

$k_0 = \tan(\Omega^{`}_p/2)/\tan(\Omega^{`}_s/2) = 0.74$

$k = k_0 = 0.74$

$k' = \sqrt{1-k}$ =0.673

by using eq. (2.40), eq. (2.41) and eq. (2.42) we find that

$D=4.9*10^7$, $q_0 = 0.05$, and $q=0.05$.

Now by applying eq. (2.39) we find that the filter order (N) is equal to 7. So the IIR elliptic filter order equal to 7.

The magnitude response of the lowpass IIR elliptic digital filter is shown in Fig. (7), and the phase response of this filter is shown in Fig. (8), which shows that the phase response of IIR elliptic filter is nonlinear.

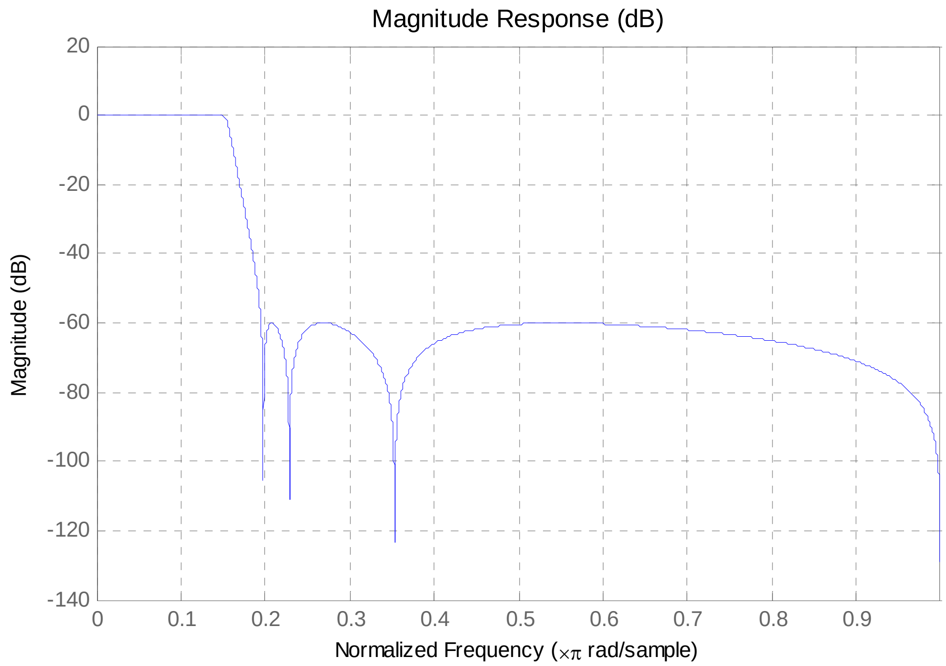


Figure (7)
Magnitude response of the lowpass elliptic IIR filter with filter order 7.

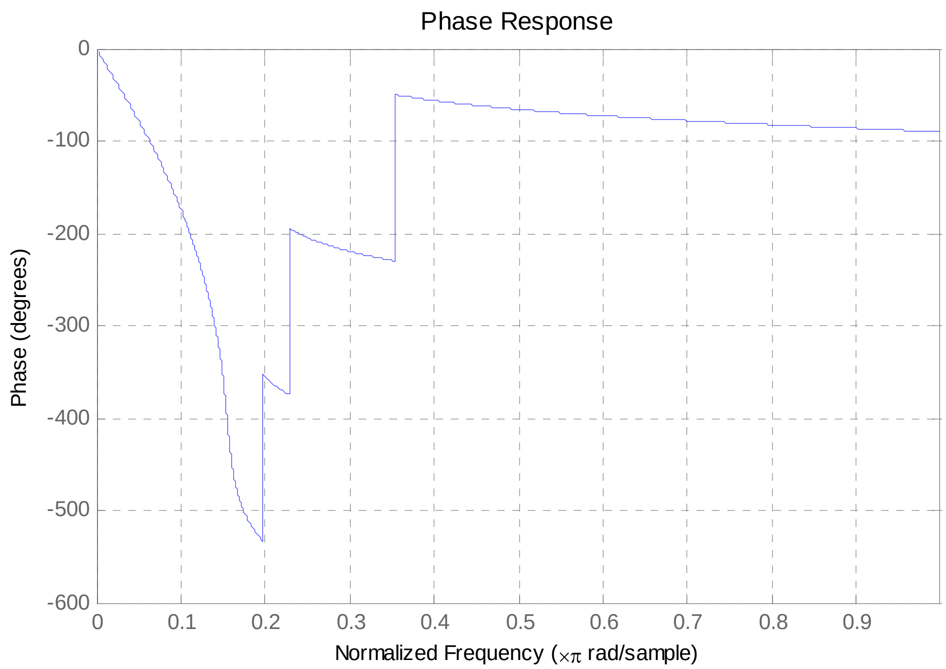


Figure (8)
The phase response of the lowpass elliptic IIR filter with filter order 7.

2.10 Comparison Between FIR Filters And IIR Filters

To perform a comparison between FIR filter and IIR filter. The FIR and IIR are designed filter with same specifications. Therefore specification of example (2.3) is considered to show this comparison and it is to be redesigned here as FIR filter.

Design Example (2.4)

Required to design a digital lowpass FIR filter with the following specifications: The passband edge frequency (w_p)=0.15л, the stopband edge frequency (w_s)=0.2 л, the passband ripple (δ_p)=0.01, and the stopband ripple (δ_s)=0.001.

Solution:

Note that the specification used here are the same specifications used to design the digital lowpass IIR elliptic filter, we found that the filter order equal to 7, and the phase response was nonlinear.

The cutoff frequency (w_c)=(w_p+w_s)/2 = 0.175 л.

E_s =-20log$_{10}$ δ_s =60dB.

The peak passband attenuation (E_p) is:

E_p=-20log_{10}(1- δ_p)=-20log_{10}(1-0.01) = 0.087dB.

By using eq.(2.17) and eq. (2.18) we find that the value of parameter (β) equal to 5.6533. The transition banwidth (Δf) equal to

Δf=(w_s-w_p)/2л =(0.2 л-0.15 л)/2 л= 0.0785

Now by using eq. (2.19) we find that the filter order (N) equal to 146.

The magnitude response of the lowpass FIR digital filter is shown in Fig. (9), from Fig. (10) we see that the phase response of the lowpass FIR digital filter is linear.

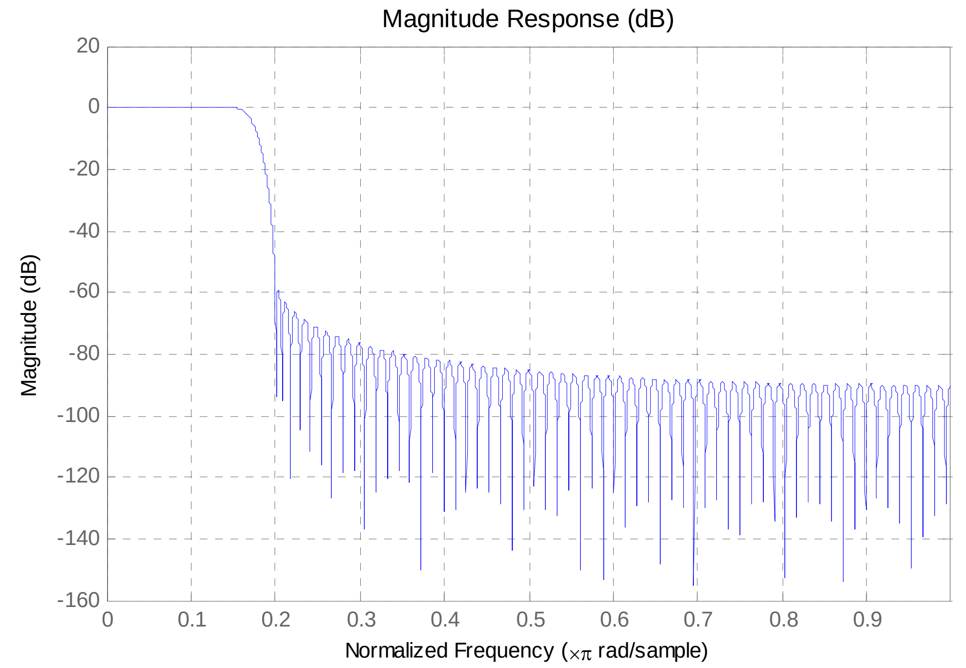


Figure (9)
The magnitude response of the lowpass FIR digital filter with *N*=146.

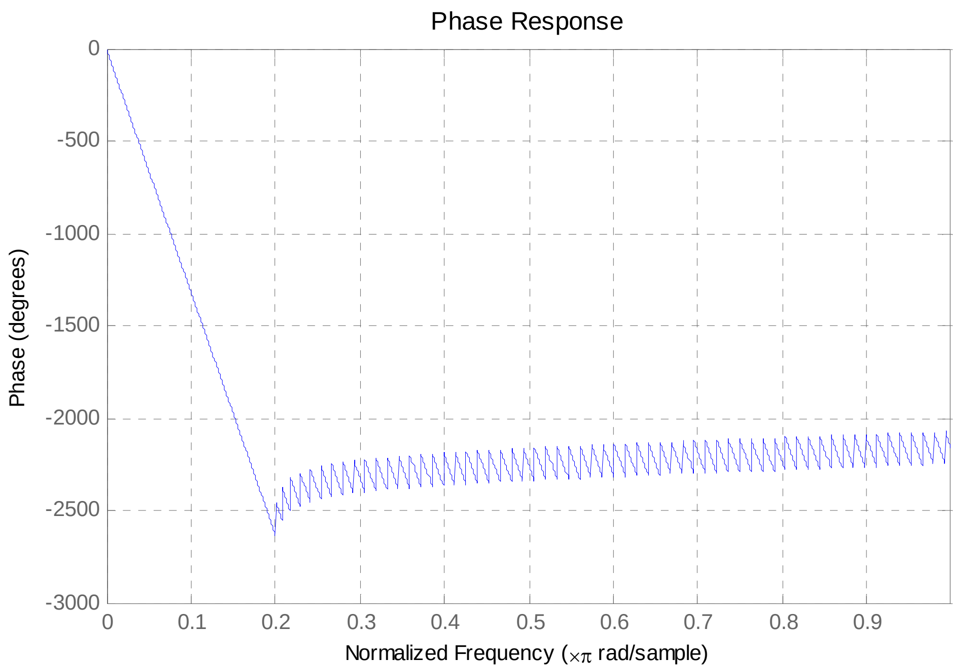


Figure (10)
The phase response of the lowpass FIR digital filter with *N*=146.

From example (2.3) and example (2.4) we see that the phase response of FIR filter is linear, while the phase response of the IIR filter is nonlinear. So the linearity of the phase response in the FIR filters is the main advantages over the IIR filters. So when the linearity of the phase response is important in the design we will use FIR filters. On the other hand if the linearity of the phase response is not important, it is sometimes preferable to use IIR filters, because an IIR filter usually requires a much lower order for the same specifications.

Note that for the same specifications the FIR digital filter using Kaiser method needs a filter order N equal to 146, on the other hand, for the same specifications the IIR digital filter needs a filter order N equals to 7, which means that the implementation of IIR digital filter is most simpler than FIR digital filter. But which types of digital filters that can be used depend on the applications that the filter can be applied to. In practice, FIR digital filters can be used when we need a linear phase response, while when the linearity of phase response is not important we can use IIR digital filter since it lower filter order which is simpler to implement and less costly.

2.11 Conclusions

There are two types of digital filters which are the non-recursive and the recursive filters. The non-recursive filter associated with digital filters whose impulse response is finite in duration (the impulse response of the digital filters is zero outside a finite interval) so it is denoted as finite impulse response (FIR) filters. While the recursive filters are associated with impulse responses whose duration are infinite and they are referred to as infinite impulse response (IIR) digital filters.

Kaiser window which is used to design a digital FIR filters is a parametric window function, which has a shape that can be determined by different parameters (which are the filter length (N+1) and the parameter (β)), and can be adjusted to satisfy given constraints on the frequency response.

As the peak ripple of FIR filters decrease, the filter order (N) will increase, and as the filter order (N) increase, the ripples get crowded closer to the cutoff frequency (w_c).

FIR filters usually have a linear phase response which means that, different frequencies will have the same delay as they pass through this filter.

The methods that can be used to design IIR filters differ from the methods that can be used to design FIR filters, since the transfer function of

IIR filter is a ratio of polynomials of z, while the transfer function of FIR filter is a polynomial of z^{-1}.

The elliptic filter is a type of Infinite Impulse Response (IIR) filters, because its response to an impulse input remains non-zero until infinite time. This comes as an improvement over Chebyshev filters, where the elliptic filter exhibits equiripple behavior in both passband and stopband. Because of the equiripple nature in both passband and stopband, the elliptic filter requires a smaller order than other filters for the same specifications [2].

An IIR filters have a non-linear phase response. which means that different frequencies will have different delay as they pass through the filter. The lost of phase linearity is attributed to the lack of symmetry of the impulse response.

The real advantage of using an FIR filter compared to an IIR filter is that the phase response is linear, meaning that all frequencies in the passband have the same delay, when they pass through the filter.

For the same specifications the filter order of the IIR digital filter is much smaller than the FIR digital filter.

Chapter Three
Fundamentals of Multirate Digital Signal Processing

3.1 Introduction

Within DSP (Digital Signal Processing) lies the word of multirate signal processing . The term multirate is used when more than one sampling rate (also called the sampling frequency) is used.

The process of increasing the sampling rate by an integer factor is called the interpolation [5] while the decreasing of the sampling rate by integer factor is called the decimation process. This chapter presents a basic multirate operations that are later used in the filter bank design.

This chapter includes the study of multirate system and multirate filter design, which include, discrete time signal processing, fundamental building blocks, sampling rate conversion, decimation/interpolation filter systems, noble identities, polyphase representation, and multirate digital filters.

3.2 Discrete-Time Signal Processing

All of the signals of interest in DSP are discrete-time sequences of either real or complex numbers. These sequences are typically obtained by

uniformly sampling a continuous-time signal, such as a voltage signal as a function of time. For example, if $x_c(t)$ denotes a continuous-time signal, then the sequence $x(n) = x_c(nT_s)$ for $n \in Z$ denotes a discrete-time signal obtained by uniformly sampling $x_c(t)$ every T_s units of time.

In order for the discrete-time signal $x(n)$ to be stored digitally on a computer, for example, the value of $x(n)$ at each n must be approximated by a value obtained from a finite set of possible values. This process is called quantization or discretization. Typically, we will assume that the number of discretization levels is large enough so as to ignore the effects of quantization.

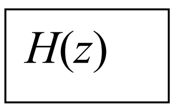
Signal processing analysis is often facilitated by considering alternative representations of discrete time signals and systems. Perhaps the most commonly used alternative representations of a discrete time signal $x(n)$ are its z-transform $X(z)$ and its Fourier transform $X(e^{j\omega})$. The z-transform is defined

$$X(z) = \sum_{n=\infty}^{-\infty} x(n).z^{-n} \tag{3.1}$$

for regions of $z \in C$ for which the above summation converges. Also, the Fourier transform $X(e^{j\omega})$ is simply $X(z)$ evaluated on the unit circle $z = e^{j\omega}$, assuming that the summation converges there. In cases where the Fourier transform $X(e^{j\omega})$ exists but the z-transform $X(z)$ does not [2], we will write $X(z)$ as a short hand notation for $X(e^{j\omega})$.

3.3 Fundamental Building Blocks

The majority of multirate DSP systems consists of three fundamental building blocks. These are the linear time-invariant (LTI) filter, the expander, and the decimator. An LTI filter, such as the one shown in Fig. (11), is characterized by its impulse response $h(n)$ (i.e., its response to the unit Kronecker delta impulse function $x(n) = \delta(n)$ [2]), or equivalently by its z-transform $H(z)$, which is called the transfer function. The system of Fig. (11) is an example of a single-input single-output (SISO) LTI system. More generally, a multiple-input multiple-output (MIMO) LTI filter, such as the one shown in Fig. (12) operates on a $r \times 1$ vector input $\mathbf{x}(n)$ to produce an $p \times 1$ vector output $y(n)$. Such a filter is characterized by its $p \times r$ impulse response matrix sequence $h(n)$ or equivalently by its $p \times r$ transfer function matrix $H(z)$.



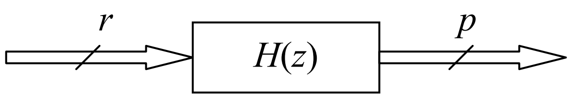
$$\sum_{k=-\infty}^{\infty}$$

$x(n)$ ——— ——— $y(n)=$ $h(k)x(n-k)$

$X(z)$ $Y(z)=H(z)X(z)$

Figure (11)
Linear time-invariant (LTI) filtering operation.

$x(n)$



$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

$X(z)$ $Y(z)=H(z)X(z)$

Figure (12)
Multiple-input multiple-output (MIMO) LTI filtering operation.

3.4 Sampling Rate Conversion

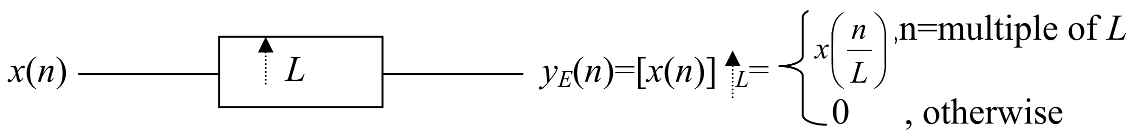
The process of digital conversion the sampling rate of a signal from a given rate F=1/T to a different rate F'=1/T' is sampling rate conversion [4], where F is the sampling rate with sampling period T and F' is the new sampling rate with sampling period T'.

The most basic operations in multirate digital signal processing are decimation and interpolation [2]. Decimator and interpolator have another names, so the decimator sometimes called a downsampler, subsampler, or compressor, so any of these names can be used to indicate to the decimator. While the interpolator sometimes called upsampler, or expander, also any of these names can be used to indicate to the interpolator.

Interpolator and decimator are used to change the rate of the signal. An L-fold expander, is used to increase the sampling rate by a factor L. The L-fold expander pad ($L-1$) zero between each sample of its input and as such, the output must operate at a rate of L times the input rate in order to maintain the proper sampling rate of the input signal. Fig. (13) shows the block diagram of L-fold expander and an example for L=2.

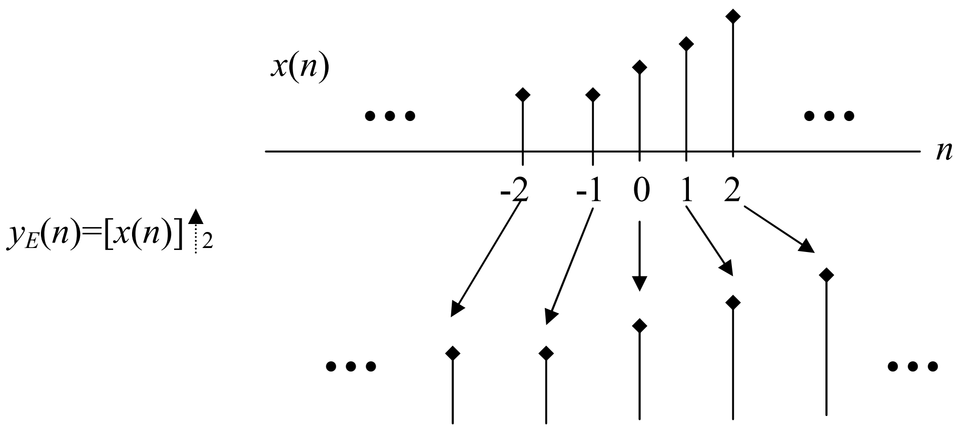
In contrast to the L-fold expander, the M-fold decimator, such as shown in Fig. (14) (a), is used to reduce the sampling rate by a factor M. The M-fold

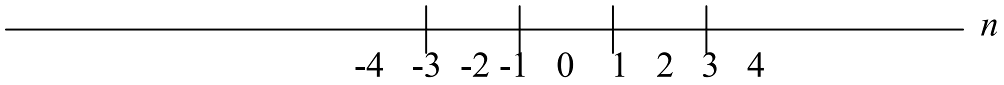
decimator only preserves every M-th sample of its input and as such, the output must operate at an M times lower rate in order to keep the proper sampling rate of the input signal. This is shown in Fig. (14) (b) for M=3.



$$y_E(n)=[x(n)]\uparrow_L=\begin{cases} x\left(\frac{n}{L}\right), & n=\text{multiple of } L \\ 0, & \text{otherwise} \end{cases}$$

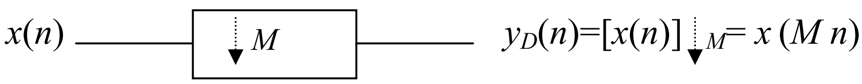
(a)



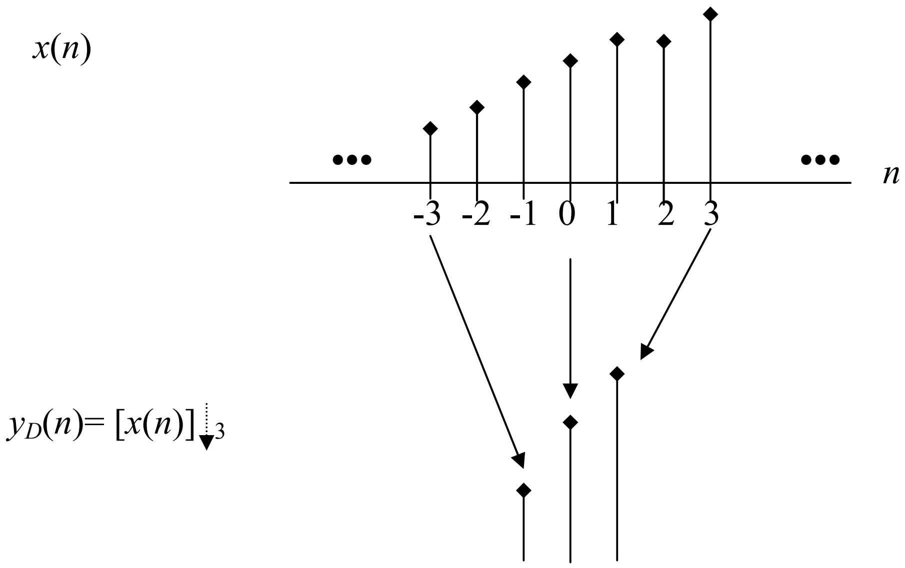


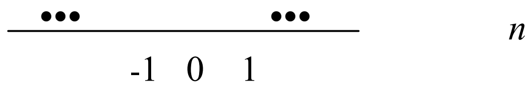
(b)

Figure (13)
L-fold expander input/output relationship: (a) Block diagram, (b) Example for $L = 2$.



(a)





(b)

Figure (14)
M-fold decimator input/output relationship: (a) Block diagram, (b) Example for $M = 3$.

Frequency or *z*-domain representations of the input/output relationships of expanders and decimators provide more insight about the nature of these rate altering building blocks. From Fig. (13)(a) and (14) (a), it can be shown [2] that we have

$$Y_E(z) = [X(z)]_{\uparrow L} = X(z^L) \qquad (L\text{-fold expander}) \qquad (3.2)$$

$$Y_D(z) = [X(z)]_{\downarrow M} = \frac{1}{M}\sum_{k=1}^{M-1} X(z^{\frac{1}{M}} W_M^K) \qquad (M\text{-fold decimator}) \qquad (3.3)$$

where

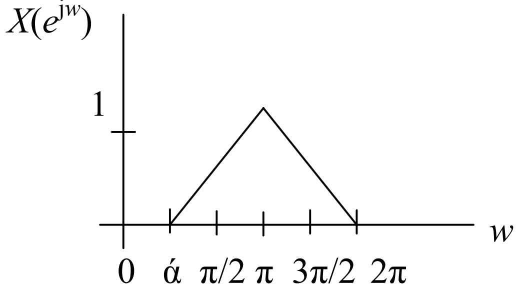
$$W_M = e^{(-2\pi j/M)} \qquad (3.4)$$

denotes the *M*-th root of unity.

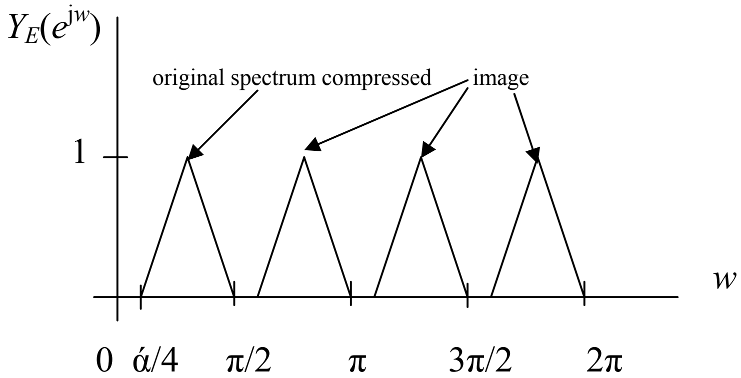
In the frequency domain, the *L*-fold expander output $Y_E(e^{j\omega})$, consists of an *L*-fold compressed version of $X(e^{j\omega})$ along with $(L - 1)$ uniformly shifted copies called images. Similarly, the *M*-fold decimator output $Y_D(e^{j\omega})$ consists of an *M*-fold stretched version of $X(e^{j\omega})$ (scaled by $1/M$) along with $(M - 1)$ uniformly shifted scaled copies called alias components. This is shown in Fig. (15) for both the expander and decimator for $L = M = 4$. The presence of $(L-1)$ images for an *L*-fold expander indicate a redundancy of a factor of *L*, where as the $(M-1)$ alias components for an *M*-fold decimator indicate a loss of information of a factor of *M*. While the original input $X(z)$ can always be recovered by its *L*-fold expanded version $Y_E(z)$, it can not in general be recovered by its *M*-fold decimated version $Y_D(z)$.

Special combinations of LTI systems and decimators/expanders can be used to parallelize/serialize discrete-time operations. One common way of parallelizing or vectorizing a scalar signal is the *M*-fold blocking system

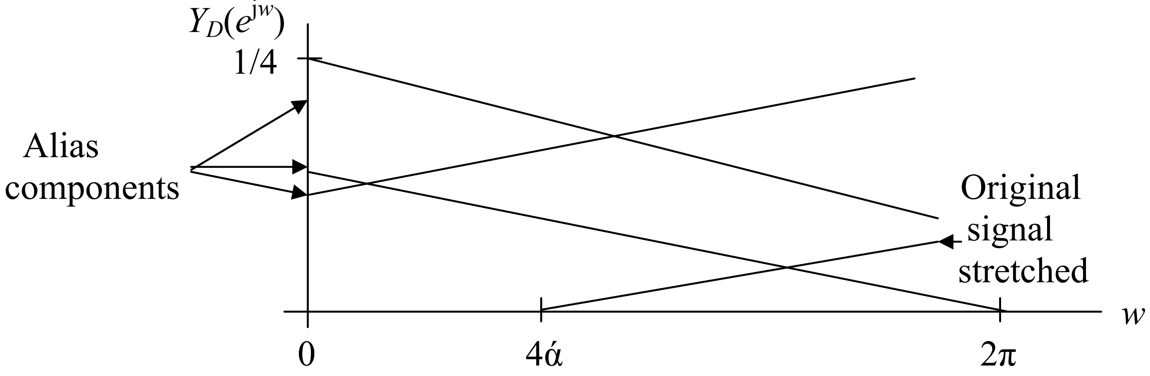
shown in Fig. (16) (a). The blocked signal vector $\mathbf{x}(n)$ is obtained by stacking every nonoverlapping block of M samples of $x(n)$ into a vector. Note that the overall rate of the blocked signal $\mathbf{x}(n)$ is the same as that of the input $x(n)$, since $\mathbf{x}(n)$ consists of M components each operating at $(1/M)$ times the input rate. The dual structure of the blocking system of Fig. (16) (a) is the L-fold unblocking system shown in Fig. (16) (b). This system is used to serialize or scalarize a vector signal input $y(n)$. The components of the input vector $y(n)$ are interlaced with each other to produce the output $y(n)$. As with the blocking system, the overall rate of the unblocking system output $y(n)$ is the same as its input $y(n)$, since $y(n)$ consists of $(1/L)$ of the components of $y(n)$ operating at L times the input rate. Blocking and unblocking systems naturally arise upon exploiting polyphase decompositions of LTI systems in conjunction with decimators and expanders, as is shown in the next section.



(a)

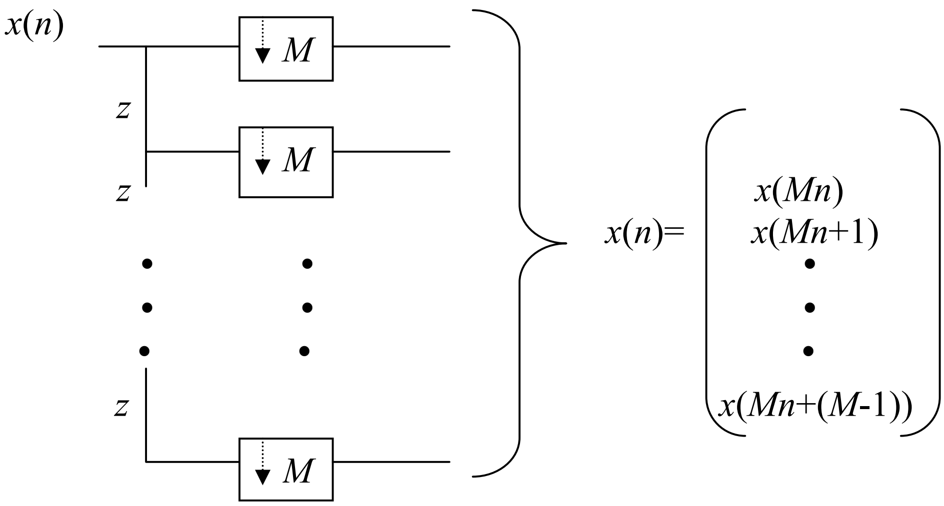


(b)

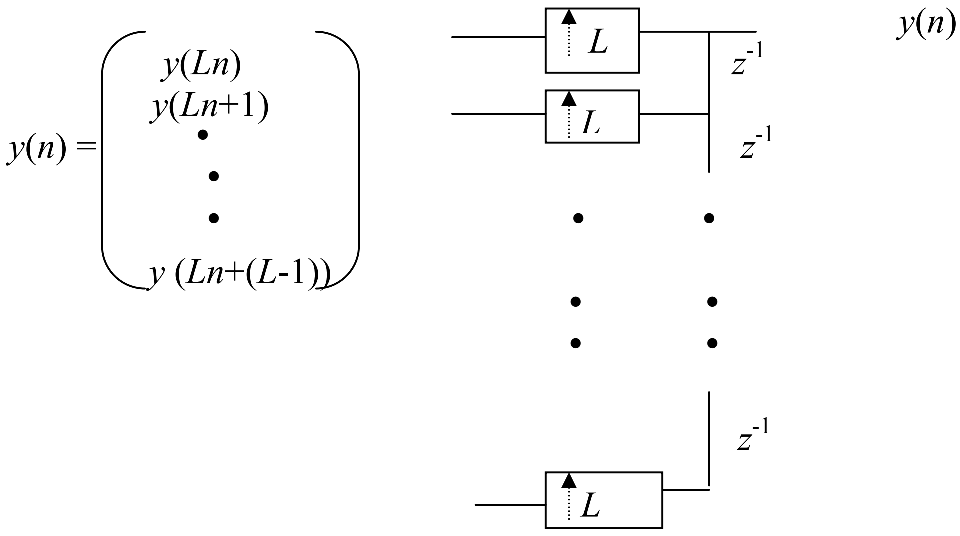


(c)

Figure (15)
(a) Given input signal spectrum $X(e^{jw})$, (b) Expanded signal spectrum $Y_E(e^{jw})$ ($L = 4$), (c) Decimated signal spectrum $Y_D(e^{jw})$ ($M = 4$).



(a)

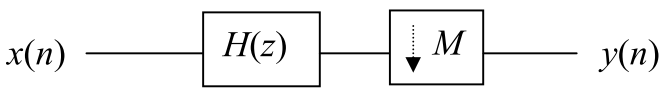


(b)

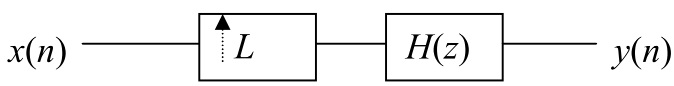
Figure (16)
(a) M-fold blocking system, (b) L-fold unblocking system.

3.5 Decimation/Interpolation Filter Systems

Remember that decimators and expanders are responsible for the undesirable phenomena of aliasing and imaging, respectively. In multirate signal processing, LTI filtering is commonly used to remove these artifacts. To remove the effects of aliasing a signal is filtered before being decimated, whereas to counteract the effects of imaging, a signal is filtered after being expanded. This is shown in Fig. (17) (a) and (b), respectively. The system of Fig. (17) (a) is known as a decimation filter system whereas that shown in Fig. (17) (b) is known as an interpolation filter system [2].



(a)



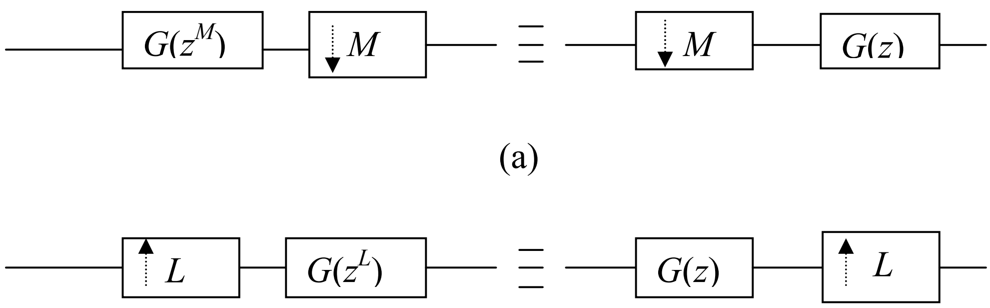
(b)

Figure (17)
(a) M-fold decimation filter system, (b) L-fold interpolation filter system.

The effects of aliasing and imaging can be attenuated or even eliminated through these systems, their current implementations are shown in Fig. (17). Note that the decimation filter system only keeps every M-th output sample and the interpolation filter system is only feeding a nonzero input to its filter at every L-th instance of time. Polyphase decompositions also can be used to design the decimation/interpolation filter systems together with the noble identities and generally will be used to design a multirate digital filter banks so we can obtain computationally efficient structures for these systems.

3.6 Noble Identities

The noble identities are very useful in the multirate systems. Note that the noble identities, when they can be used, provide computationally efficient implementations, since they give us a way to filter after decimating and before expanding. However, note that the decimator/expander noble identities only apply when the original filter system is a function of z^M or z^L, respectively as shown in Fig. (18).



(a) (b)

Figure (18)
(a) Decimator noble identity, (b) Expander noble identity.

3.7 Polyphase Representation [2,5]

Consider a filter with impulse response $h[n]$ and frequency response:

$$H(z) = \sum h[n].z^{-n}$$

(3.5)

We show first the two-channel polyphase representation. The previous equation can be written equivalently as

$$H(z) = \sum h[2n].z^{-2n} + z^{-1}\sum h[2n+1].z^{-2n}. \tag{3.6}$$

define

$$E_0(z) = \sum h[2n].z^{-n}. \tag{3.7}$$

and

$$E_1(z) = \sum h[2n+1].z^{-n}. \tag{3.8}$$

So that, $H(z)$ can be represented as

$$H(z) = E_0(z^2) + z^{-1}E_1(z^2) \tag{3.9}$$

This is a representation with two phases. In general, for a M-channel filter bank we have

$$H(z) = \sum_{n\in Z} h[nM].z^{-nM} + z^{-1}\sum_{n\in Z} h[nM+1].z^{-nM} + ... + \\ z^{-(M-1)}\sum_{n\in Z} h[nM+M-1].z^{-nM}. \tag{3.10}$$

or

$$H(z) = \sum_{l=0}^{M-1} z^{-l}.E_l(z^M). \tag{3.11}$$

$$E_l(z) = \sum e_l[n].z^{-n}. \tag{3.12}$$

$$e_l[n] = h[Mn+1], \qquad 0 \le l < M. \tag{3.13}$$

3.8 Multirate Digital Filters

As digital systems may evolve into more advanced systems. It is important to have some techniques which facilitate communication between systems with different sampling frequencies. These techniques are called multirate techniques. Digital signal processing systems which make use of multirate techniques are called multirate systems. The same techniques can also be used to improve the efficiency of the signal processing by changing sampling frequency internally while input and output frequencies remain unchanged. In this case, the digital signal processing system works at a lower sampling frequency than the input/output frequency, resulting in reduced computational workload of the whole system.

Multirate filters are digital filters that operate with one more sample rate change embedded in the signal processing architecture. There are two reasons to change the sampling rate, the first is the performance, and the second is the cost. Multirate system often perform a processing task with improved performance characteristics while simultaneously offering that performance at significantly lower cost than traditional approaches[9]. Multirate Filter Designs use the basic properties of decimation and interpolation in the implementation of the filter.

To realize multirate systems, digital filters together with upsamplers and downsamplers, are used. There are two different types of digital filters, as has been indicated, FIR (Finite Impulse Response) and IIR (Infinite Impulse Response) filters.

3.8.1 FIR Decimator and Interpolator

This section explains the design of the FIR digital filters that apply to the design of multirate systems. The same techniques that can be used to design the FIR digital filters are also can be used to design the FIR interpolator and decimator filters. Kaiser window will be used to design these filters, which come as a continuation to the previous chapter.

FIR interpolator is used when there is a need to increase the sampling rate by an integer number (L), and this can be accomplished by inserting (L-1) zeros between a successive value of the signal. On the other hand FIR decimator is used when there is a need to decrease the sampling rate by an integer number(M), and this can be accomplished by preserving every M-th sample of its input signal.

The filter order of the FIR interpolator (N') is equal to the order of the original FIR filter (N) times the interpolator factor (L) which is given by

$$N' = N*L \tag{3.14}$$

while the filter order of the FIR decimator (N') equal to the order of the original FIR filter (N) divided by the decimator factor (M) which is given by

$$N' = N/M \tag{3.15}$$

FIR filters commonly have a linear phase response, also FIR interpolator and decimator filters have a linear phase response which means that different frequencies have the same delay when passing through the filter, so that FIR interpolator and decimator filters are used in some applications where the linearity of the phase response are required.

Design Example (FIR Interpolator Filter) (3.1)

Design an interpolator that increase the input sampling rate by a factor of 4 by using Kaiser window technique. Suppose the design specifications are: The passband edge frequency (w_p)=0.2π, the stopband edge frequency (w_s)=0.3 π, the minimum stopband attenuation (E_s)=40dB, and the passband ripple ($\delta 1$) = the stopband ripple ($\delta 2$).

Solution:

This example is actually has the same specification given in example (2.1), in the last example we found that the filter order (N) equal to 45, while we find here that the filter order (N') equal to 180 which equal to interpolator factor times the filter order (N), so N' = 4*45 = 180.

The magnitude response of the FIR interpolator filter for L=4 is shown in Fig. (19). From Fig. (20) we see that the phase response of the FIR interpolator filter is linear phase.

From Fig (19) we see that the passband edge frequency (w_p) and the stopband edge frequency (w_s) of the FIR interpolator are reduced by the interpolator factor, so the passband edge frequency (w_p) of the FIR interpolator equal to the passband edge frequency (w_p) of the original FIR filter divided by the interpolator factor, also the stopband edge frequency (w_s) of the FIR interpolator equal to the stopband edge frequency (w_s) of the original FIR filter divided by the interpolator factor. To design an FIR filter with the same frequencies edges (passband edge frequency and stopband edge

frequency) of the FIR interpolator we need a filter order higher than 180 which will increase the implementation complexity and the cost of the FIR filter design, therefore FIR interpolator is used also to reduce the implementation complexity and the cost of the filter design.

The passband edge frequency (w_p) of the FIR interpolator equal to $0.2\pi/4$ which equal to $0.05\ \pi$, while the stopband edge frequency (w_s) of the FIR interpolator equal to $0.3\pi/4$ which equal to 0.075π.

From Fig. (19) We see that the transition width of the FIR interpolator filter decrease by the interpolator factor, while the overall bandwidth remains the same as the original FIR filter.

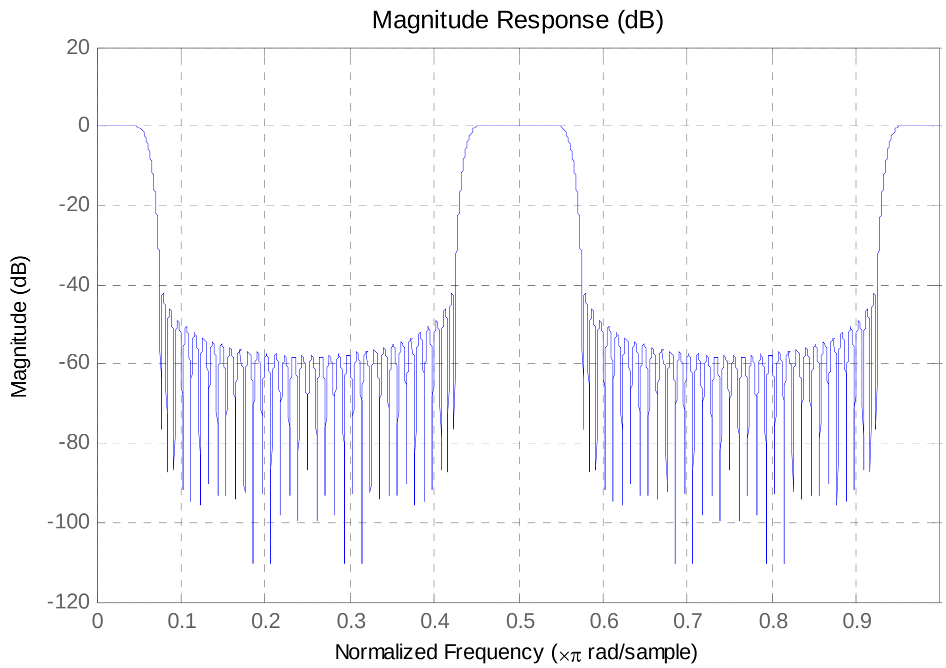


Figure (19)
The magnitude response of the FIR interpolator filter with L=4.

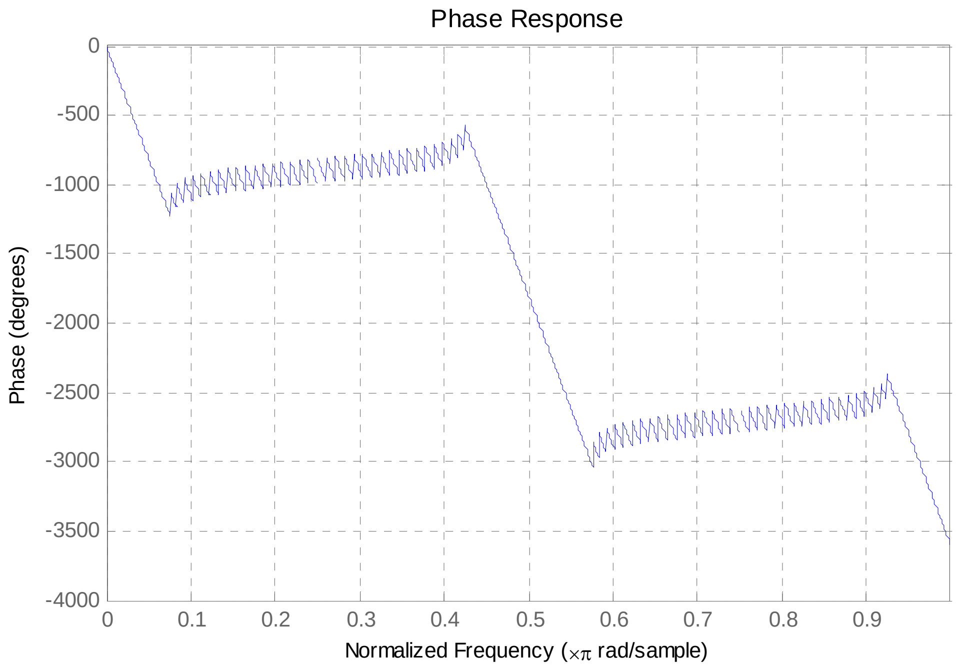


Figure (20)
The phase response of the FIR interpolator filter with L=4.

Design Example (FIR Interpolator Filter) (3.2)

Design an interpolator that increase the input sampling rate by a factor of 8 by using Kaiser window technique. Suppose that the design specifications are the same as in the previous example.

Solution:

We find that the filter order of the original FIR digital filter equal to 45, while the order of the FIR interpolator filter which has the same specifications equal to 360, so $N' = 8*45 = 360$.

The passband edge frequency (w_p) of the FIR interpolator equal to $0.2\pi/8$ which equal to $0.025\ \pi$, while the stopband edge frequency (w_s) of the FIR interpolator equal to $0.3\pi/8$ which equal to 0.0375π.

The magnitude response of the FIR interpolator filter for L=8 is shown in Fig. (21). Also the phase response of the FIR interpolator filter is shown in Fig. (22) which is linear.

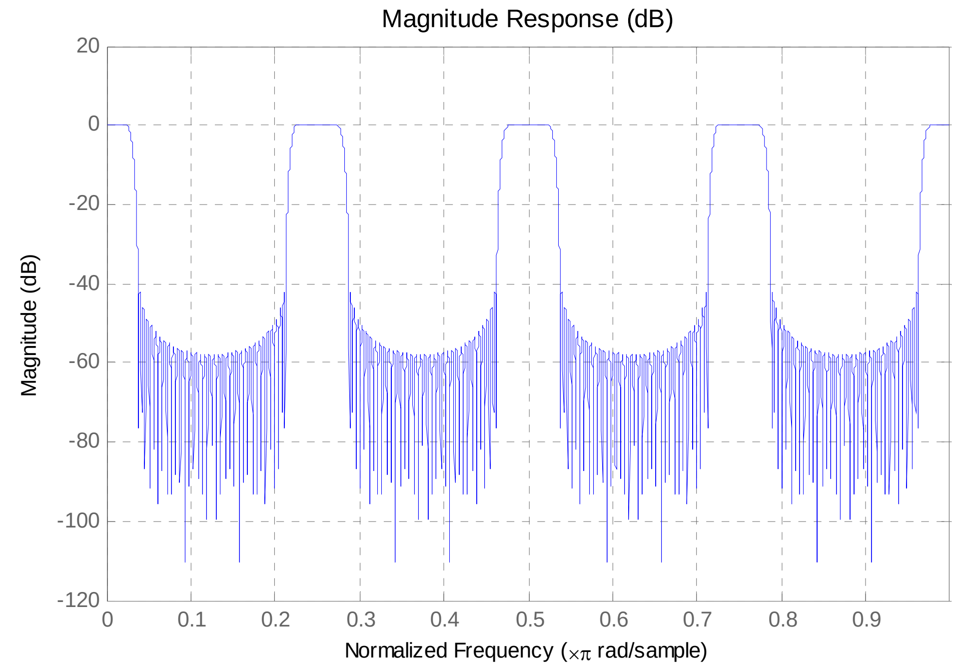


Figure (21)
The magnitude response of the FIR interpolator filter with *L*=8.

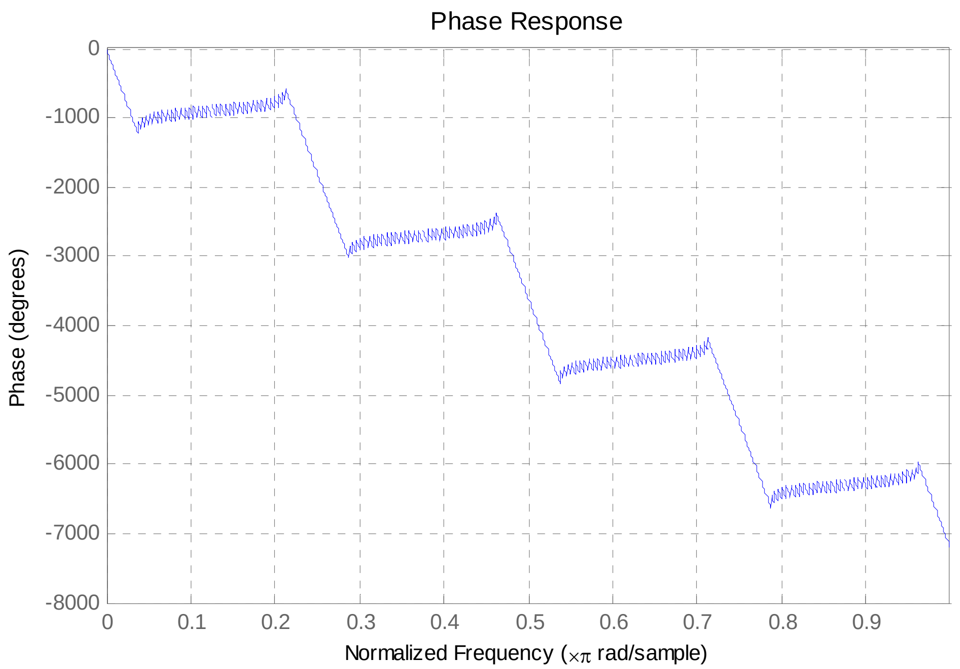


Figure (22)
The phase response of the FIR interpolator filter with *L*=8.

We see that as the interpolator factor increase, the filter order increase, while the transition width decrease by an interpolator factor as show in Fig. (19) and (21), while the overall bandwidth of the FIR interpolator filter remains the same as the original FIR filter.

Design Example (FIR Decimator Filter) (3.3)

Design an decimator that decrease the input sampling rate by a factor of 3 by using Kaiser window technique. Suppose the design specifications are the same as the previous example.

Solution:

We find that the filter order (N') of the FIR decimator equal to 15, while the filter order of the FIR filter which has the same specifications equal to 45, which means that the order of the FIR decimator equal to the order of the FIR filter which has the same specifications divided by the decimator factor.

N' = 45/3 = 15.

Note that the passband edge frequency (w_p) and the stopband edge frequency (w_s) of the FIR decimator are increased by the decimator factor. Therefore the passband edge frequency (w_p) of the FIR decimator equal to 0.2*л**3 which equal to 0.6 *л* and the the stopband edge frequency (w_s) of the FIR decimator equal t0 0.3*л**3 which equal to 0.9*л*. We see that the transition width of the FIR decimator filter increase by the decimator factor.

The magnitude response of the FIR decimator filter for *M*=3 is shown in Fig. (23). The FIR decimator filter has a linear phase response as shown in Fig. (24).

Note that there are an error in the passband attenuation and this error is due to the downsampling or decimating process. This error also effects to the stopband attenuation.

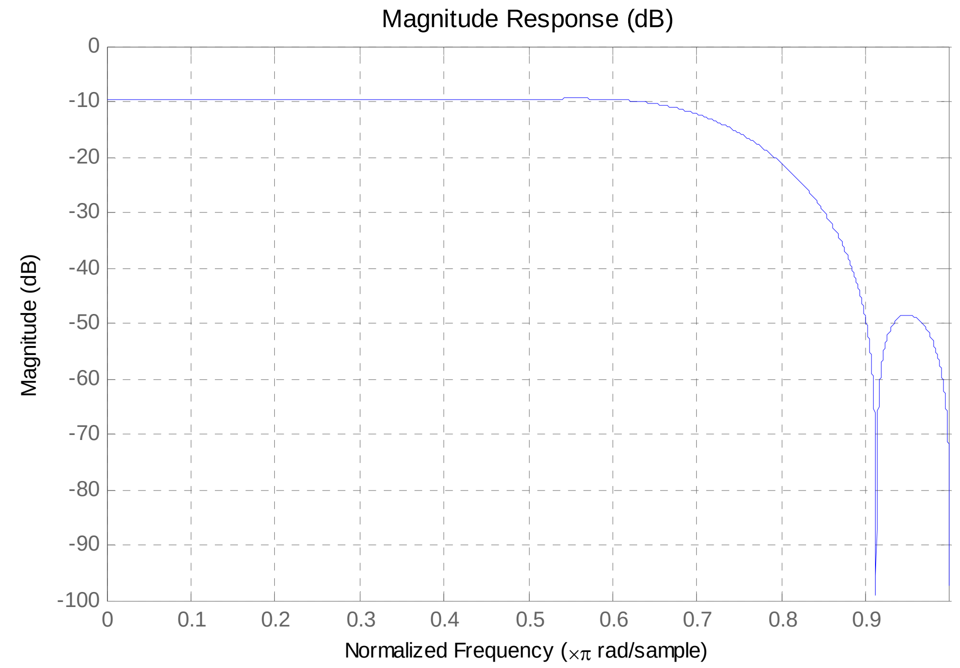


Figure (23)
The magnitude response of the FIR decimator filter for *M*=3.

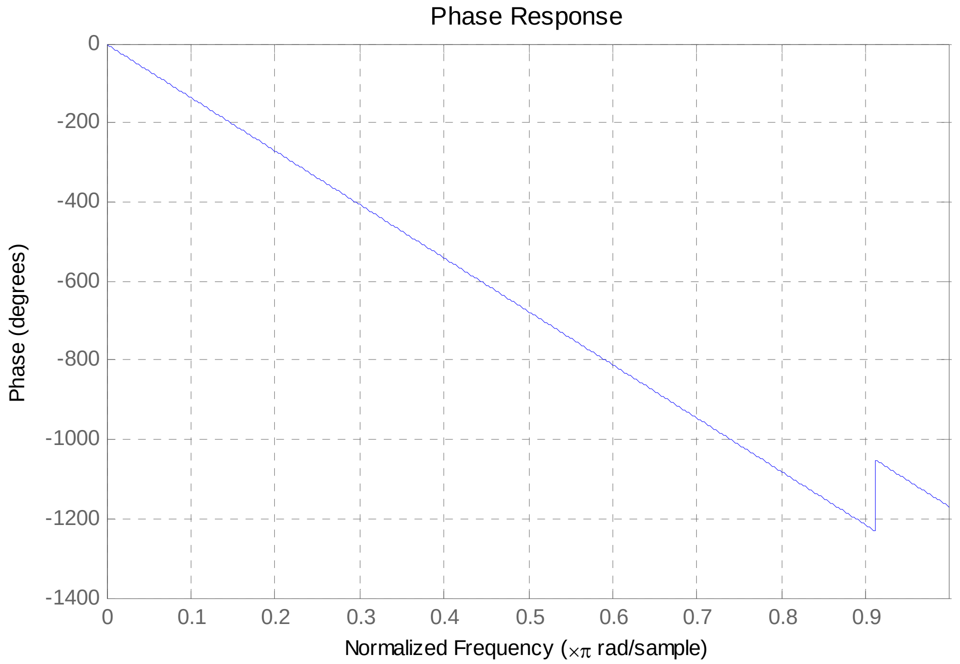


Figure (24)
The phase response of the FIR decimator filter with *M*=3.

3.8.2 IIR Decimator and Interpolator

The same techniques that can be used to design the IIR digital filters can also be used to design the IIR interpolator and decimator filters. The design of the IIR elliptic filters that apply to the design of multirate systems, will be studied here which come as a continuation to the previous chapter.

As indicated in the previous section, the interpolator filters are used to increase the sampling rate by an integer factor *L*, while the decimator filters are used to decrease the sampling rate by an integer factor *M*, regardless which type of the digital filters are used, but it depends on the applications that we need to obtain it, such as FIR digital filter are used when we need a linear phase response, on the other hand IIR digital filters are used when no need to linear phase response.

The filter order of the IIR interpolator ($N^{'}$) equal to the order of the original IIR filter (N) times the interpolator factor (L) which is given by

$$N^{'} = N*L \tag{3.16}$$

while the filter order of the IIR decimator ($N^{'}$) equal to the order of the original IIR filter (N) divided by the decimator factor (M) which is given by

$$N^{'} = N/M \tag{3.17}$$

Design Example (IIR Interpolator Filter) (3.4)

Required to design an IIR elliptic interpolator that increase the input sampling rate by a factor of 5, use the same specification in example (2.3) .

Solution:

In the previous example we found that the original IIR filter order (N) equal to 7, while we find here that the filter order ($N^{'}$) equal to 35, which equal to interpolator factor times the filter order (N).

$N^{'} = 7*5 = 35$.

The magnitude response of the IIR interpolator filter is shown in Fig. (25). From Fig. (26) we see that the phase response of the IIR elliptic interpolator is nonlinear.

From Fig (25) we see that the passband edge frequency (w_p) and the stopband edge frequency (w_s) of the IIR interpolator are reduced by the interpolator factor, so the passband edge frequency (w_p) of the IIR interpolator equal to the passband edge frequency (w_p) of the original IIR filter divided by

the interpolator factor, also the stopband edge frequency (w_s) of the IIR interpolator equal to the stopband edge frequency (w_s) of the original IIR filter divided by the interpolator factor.

The passband edge frequency (w_p) of the IIR interpolator equal to $0.15\pi/5$ which equal to $0.03\ \pi$, while the stopband edge frequency (w_s) of the IIR interpolator equal to $0.2\pi/4$ which equal to 0.04π.

From Fig. (25) We see that the transition width of the IIR interpolator filter decrease by the interpolator factor, while the overall bandwidth remains the same as the original IIR filter.

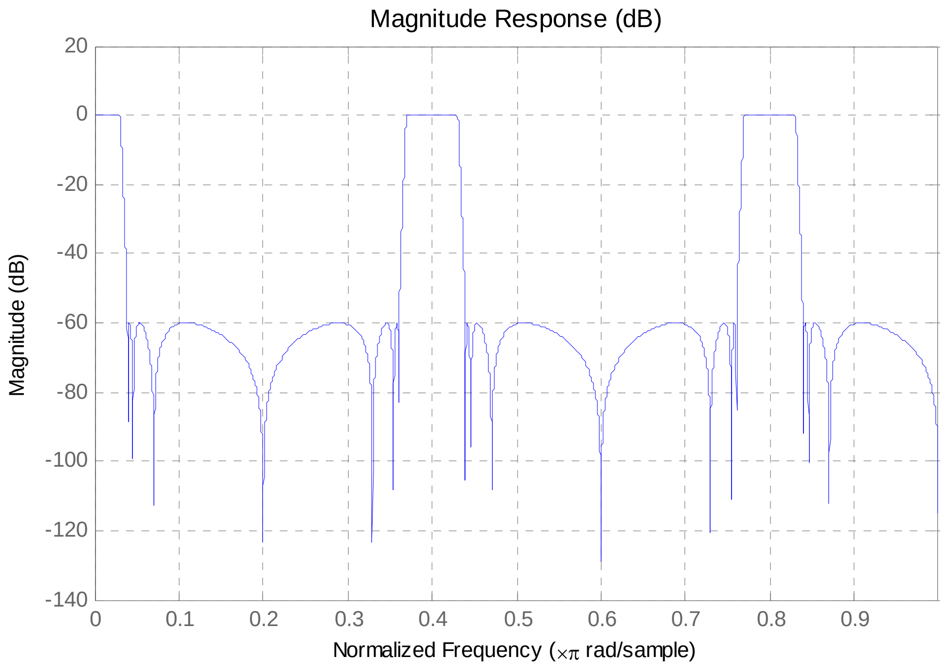


Figure (25)
The magnitude response of the IIR elliptic interpolator filter with L=5.

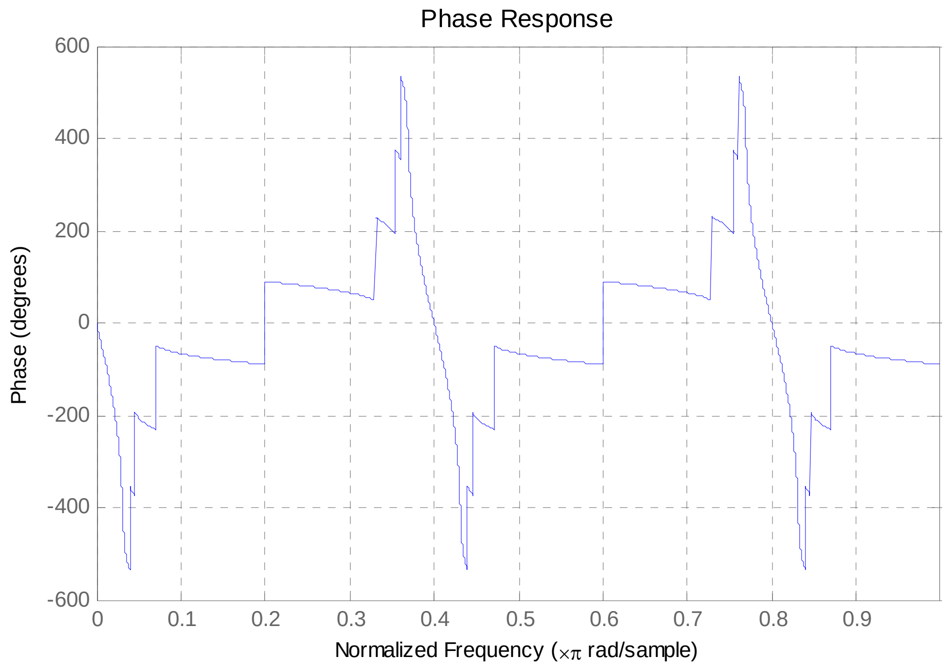


Figure (26)
The phase response of the IIR elliptic interpolator filter with L=5.

Design Example (IIR Interpolator Filter) (3.5)

Required to design an IIR elliptic interpolator that increase the input sampling rate by a factor of 4, use the same specification in example (2.3)

Solution:

In the previous example we found that the original IIR filter order (N) equal to 7, while we find here that the filter order (N') equal to 28, which equal to interpolator factor times the filter order (N) so $N' = 7*4 = 28$.

The magnitude response of the IIR interpolator filter is shown in Fig. (27). From Fig. (28) we see that the phase response of the IIR elliptic interpolator is nonlinear.

From Fig (27) we see that the passband edge frequency (w_p) and the stopband edge frequency (w_s) of the IIR interpolator are reduced by the interpolator factor, so the passband edge frequency (w_p) of the IIR interpolator equal to the passband edge frequency (w_p) of the original IIR filter divided by the interpolator factor, also the stopband edge frequency (w_s) of the IIR interpolator equal to the stopband edge frequency (w_s) of the original IIR filter divided by the interpolator factor.

The passband edge frequency (w_p) of the IIR interpolator equal to 0.15л/4 which equal to 0.0375 л, while the stopband edge frequency (w_s) of the IIR interpolator equal to 0.2л/4 which equal to 0.05л.

From Fig. (27) We see that the transition width of the IIR interpolator filter decrease by the interpolator factor, while the overall bandwidth remains the same as the original IIR filter.

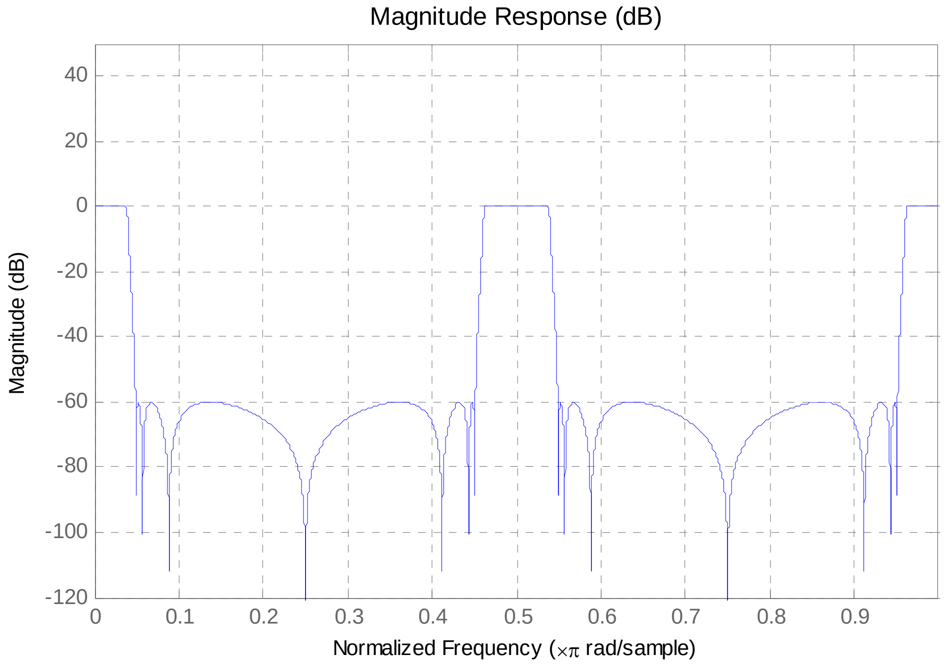


Figure (27)
The magnitude response of the IIR elliptic interpolator filter with *L*=4.

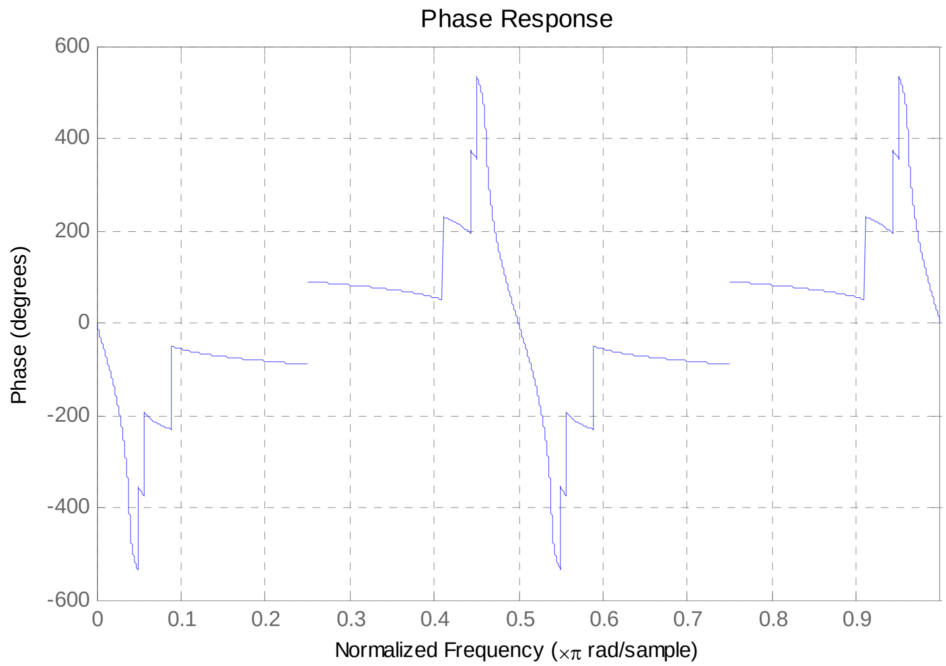


Figure (28)
The phase response of the IIR elliptic interpolator filter with *L*=4.

3.9 Conclusions

The most basic operations in multirate digital signal processing are decimation and interpolation. Interpolator and decimator are used to change the rate of the signal. An interpolator, is used to increase the sampling rate by a factor *L*. The interpolator pad *(L-1)* zero between each sample of its input signal, so the output must operate at a rate of the interpolator factor (*L*) times the input rate in order to maintain the proper sampling rate of the input signal. On the other hand, the decimator is used to decrease the sampling of its input signal. The decimator only preserves every *M*-th sample of its input and as such, the output must operate at an *M* times lower rate in order to keep the proper sampling rate of the input signal.

In conclusion, it is possible to design very efficient FIR and IIR filters using multirate design methods. The only real disadvantage is the complexity of the implementation and design unless an automated design program is available for use.

The lowpass filter can be designed by using the single rate structure where the sampling frequency is unchanged during the signal processing or by using the multirate structure where the sampling frequency is changed during the signal processing.

The same techniques that can be used to design the FIR digital filters are also can be used to design the FIR interpolator and decimator filters. Also

the same techniques that can be used to design the IIR digital filters are also can be used to design the IIR interpolator and decimator filters.

The filter order of the FIR or IIR interpolator (N') equal to the order of the original FIR or IIR filter (N) times the interpolator factor (L), while the filter order of the FIR or IIR decimator (N') equal to the order of the original FIR or IIR filter (N) divided by the decimator factor (M).

The passband edge frequency (w_p) and the stopband edge frequency (w_s) of the FIR and IIR interpolator are reduced by the interpolator factor. On the other hand the passband edge frequency (w_p) and the stopband edge frequency (w_s) of the FIR and IIR decimator are increase by the decimator factor.

FIR interpolator and decimator filters have a linear phase response which means that different frequencies have the same delay when passing through the filter, so that FIR interpolator and decimator filters are used in some applications where the linearity of the phase response are required. While IIR interpolator and decimator filters have a nonlinear phase response which means that different frequencies have the different delay when passing through the filter, so that FIR interpolator and decimator filters are used in some applications where the linearity of the phase response are not required.

Chapter Four
Multirate Digital Filter Banks Design

4.1 Introduction

Filter bank is a collection of filters, with a common input, or output, signal. Multirate filter banks are special cases of multirate systems [2, 4]. In multirate systems, digital signals are processed using more than one sampling rate, where the sampling rate is kept as low as possible, which leads to more efficient processing. Multirate techniques are for example used in filters with stringent requirements and low complexity [9]. Sampling of analog signals and lowering the sampling rate of digital signals by decimation introduces an undesired effect, known as aliasing. In the design of filter banks, aliasing may be minimized or cancelled. Many design techniques for different filter banks are proposed during the years. Key developments in this field include two-channel Quadrature Mirror Filter (QMF) Banks [10,11], the polyphase representation [12, 13,14], and the extension from two to M channels [13,15] (which will be studied in more detail with aids of the polyphase representation since it is a general case (M-channel filter banks)). It is shown in [10] that aliasing may be fully cancelled in the two-channel QMF synthesis filter bank.

Filter bank have been of great interest in a number of signal processing applications, such as transmultiplexing, speech processing, audio/video signal

compression, signal detection and spectral analysis. The steadily growing interest in filter banks, is mainly due to the wide range of applications where filter banks have successfully been applied.

The main idea of using multirate filter banks, is the ability of the system to separate in the frequency domain, the signal under consideration into two or more signals, or to compose two or more different signals into a single signal.

There are different kinds of structures of filter banks exist, for example the tree structure and the parallel structure [16]. The parallel structure is more general and favorable because of its simplicity.

A filter bank is a set of filters operating in parallel, either in an analysis mode with a single input and multiple outputs or in a synthesis mode with multiple inputs and a single output, An analysis filter bank is shown on the left side in Fig. (29) (filters $h_0(n)$, $h_1(n)$,……………, $h_{M-1}(n)$), is used to decompose a given signal into M components, typically referred to as subband signals. In most cases, these subband signals are bandlimited, therefore down sampled before subsequent use (transmission over a communication channel or storage). The down sampling factor in general may differ from channel to channel based on the characteristics of the analysis filter employed in that channel.

A synthesis filter bank, shown on the right side of Fig. (29) (filters $f_0(n), f_1(n)$,………., $f_{M-1}(n)$) enables the merging of several input signals into a higher rate signal. In principle, this operation consists of upsampling M signals to a common higher sampling rate and applying the upsampled signals to suitable filters followed by an addition of the filtered signals.

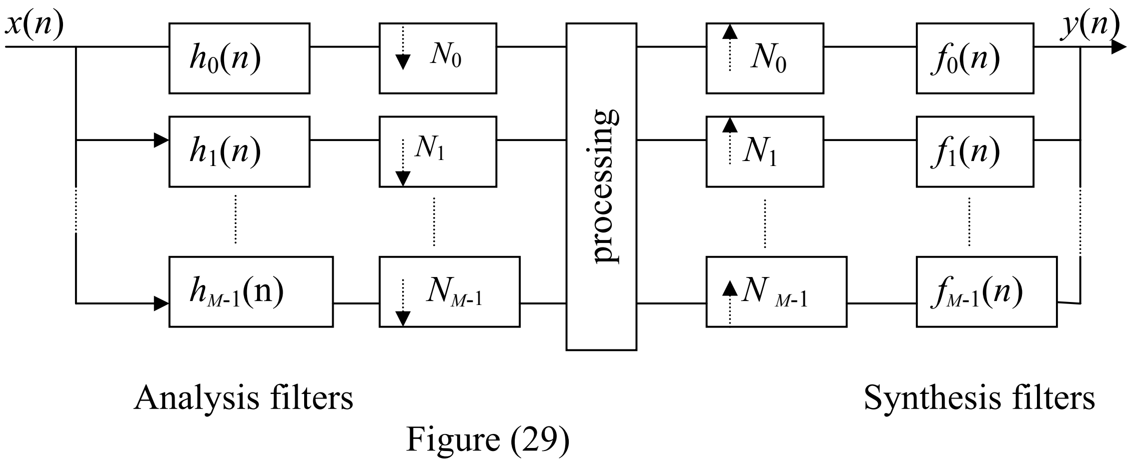


Figure (29)
A general multirate digital filter bank system.

So the filter bank system consists of four parts which are:

1) The analysis filters, which consist of a set of filters, arranged in a parallel bank as shown in Fig. (29).
2) The synthesis filters, which consist of a set of filters, arranged in a parallel bank as shown in Fig. (29).
3) The decimation factor ($\downarrow N_i$).
4) The interpolation factor ($\uparrow N_i$).

Quadrature mirror filter (QMF) is a special case of the *M*-channel multirate digital filter banks, where the number of channels are two. Therefore we have two analysis filters which are typically complementary lowpass and highpass filters that mirror each other about a digital frequency value which corresponds to one fourth the sampling frequency[2]. As a result this system is called quadrature mirror filter (QMF). Also there are two synthesis filters which are complementary lowpass and highpass. The same methods that are used to design the *M*-channel multirate digital filter banks can be use to design the quadrature mirror filter (QMF).

4.2 Perfect Reconstruction

If the filters are ideal, no aliasing occurs and perfect reconstruction can be obtained trivially. However, this is not the case in practice, so careful choices of analysis filters ($H_m(z)$) and synthesis filters ($F_m(z)$) need to be made. The most general perfect reconstruction property can be defined as follows:

A filter bank is said to have perfect reconstruction if its output signal is a pure time-delayed version of its input signal, such that the overall system must satisfy the perfect reconstruction property [17,18,19,20,21,22].

4.3 Polyphase Representation

The filter bank structure in Fig. (29) is not efficient, since all filters operate at high sampling rate. The noble identities hold the keys to simplify the filter bank implementation. To apply these relationships, it is necessary to define the polyphase format of a filter.

Consider a filter h_m of an *M*-channel filter bank with

$$H_m(z) = \sum_{n=-\infty}^{\infty} h[n].z^{-n} \tag{4.1}$$

We can rearrange all the filter coefficients into M groups, called polyphase components, as follows

$$H_m(z) = \sum_{l=0}^{M-1} E_{m,l}(z^M).z^{-l} \tag{4.2}$$

where the l-th polyphase component of $H_m(z)$ is given by

$$E_{m,l}(z) = \sum_{n=-\infty}^{\infty} h_m[nM+l].z^{-n} \tag{4.3}$$

Therefore the analysis bank can be written as

$$\begin{bmatrix} H_0(z) \\ \vdots \\ H_{M-2}(z) \\ H_{M-1}(z) \end{bmatrix} = \overbrace{\begin{bmatrix} E_{0|0}(z^M) & \dots & E_{0|M-1}(z^M) \\ \vdots & & \vdots \\ E_{M-1|0}(z^M) & \dots & E_{M-1|M-1}(z^N) \end{bmatrix}}^{\mathbf{E}(z^M)} . \begin{bmatrix} 1 \\ \vdots \\ z^{-(M-1)} \end{bmatrix} \tag{4.4}$$

where $\boldsymbol{E(z)}$ is called the analysis polyphase matrix.

$$e(z) = [1, z^{-1}, z^{-2}, \dots\dots\dots, z^{M-1}] \tag{4.5}$$

where $e(z)$ is called the delay chain.

So the analysis filter can be obtained now by using the Type I polyphase which can be expressed in the form

$$H_m(z) = \sum_{l=0}^{M-1} E_{m,l}(z^M).z^{-l} \quad \text{(Type I polyphase)} \tag{4.6}$$

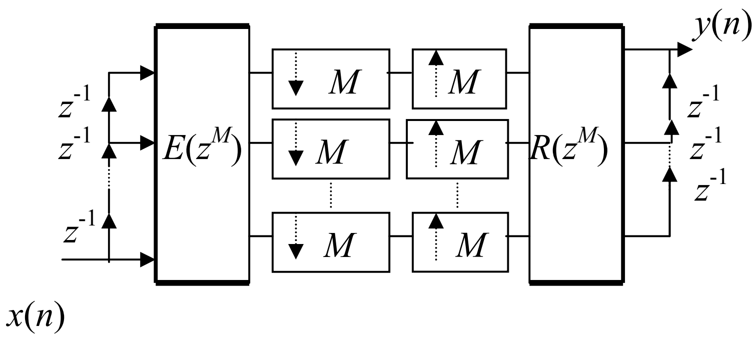
Similarly, the synthesis bank can be written as

$$\begin{bmatrix} F_0(z) \\ \vdots \\ F_{M-2}(z) \\ F_{M-1}(z) \end{bmatrix} = \left[z^{-(M-1)} z^{-(M-1)} \dots\dots\dots 1\right] \overbrace{\begin{bmatrix} R_{0|0}(z^M) & \dots & R_{0|M-1}(z^M) \\ \vdots & & \vdots \\ R_{M-1|0}(z^M) & \dots & R_{M-1|M-1}(z^N) \end{bmatrix}}^{R(z^M)} . \tag{4.7}$$

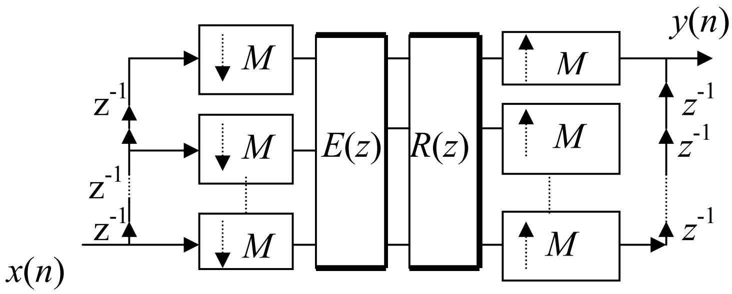
where $R(z)$ is called the synthesis polyphase matrix.
Also the synthesis filters can be expressed by using Type II polyphase as follow

$$F_m(z) = \sum_{l=0}^{M-1} z^{-(M-1-l)} R_{m,l}(z^M) \qquad \text{(Type II polyphase)} \tag{4.8}$$

Thus the M-channel filter bank depicted in Fig. (29) can be represented by Fig. (30) (a). The noble identities can then be applied to simplify Fig. (30) (a) into Fig. (30) (b), which is called the polyphase representation. It is more efficient than Fig. (29) since now the filters are operating at lower rate.



(a)



(b)

Figure (30)

Equivalent polyphase representations of an M-channel filter bank.

Notice that the m-th row of $E(z)$ contains the polyphase components of $H_m(z)$, whereas the polyphase components of $F_m(z)$ are in the m-th column of $R(z)$. The combination of the delay chain and the decimators at the analysis stage works as a serial-to-parallel converter, whereas the expanders and the delay chain at the synthesis side perform the parallel-to-serial conversion.

We see that the type I polyphase is used to find the analysis filters group, while the type II polyphase is used to find the synthesis filters group.

So that, the polyphase representation is one of the most important methods that can be used to design the filter banks systems [12,13,14,15,23,24,25]. In this chapter the polyphase structure design method is adopted as a design strategy for the filter bank system.

4.4 Multirate Digital Filter Bank Design

In order to apply the design method, a number of parameters needs to be set. The most important parameter is the number of channels. The length of the analysis and synthesis filters (which is equal to the filter order plus one) and in this design method the filter length must not exceed the number of channel, but it must equal the number of channels, in order to obtain efficient polyphase implementation, the decimator, and interpolator factor are then chosen which should have the same value and must equal to the number of channels.

Filter banks can be divided into two classes based on the way the signal is blocked and sent though the channels: uniform filter banks when all the channels are downsampled by the same factor and non-uniform filter banks otherwise. Uniform filter banks are the more common in practice, as they are easier both to analyze and to implement than the non-uniform ones. Therefore the design here is concerned with uniform filter banks.

Uniformly filter banks have been of a special interest because of there simplicity and efficient implementation. In this thesis a design method for analysis and synthesis of uniformly filter banks are presented, using polyphase implementation method.

4.4.1 Uniform Digital Filter Bank

Uniform filter banks are a collection of filters where the sampling rate, and thus the decimation factors, in all the channels are the same.

A uniform M-channel analysis filter bank is a structure consisting of analysis filters, $H_m(z)$, $m = 0, . . . , M - 1$, and decimators with decimation factor D. The task of the analysis filter bank is to transform an input signal

$X(z)$ to a set of M subband signals $X_m(z)$, which are sampled at a lower rate, see Fig. (31).

A corresponding M-channel synthesis filter bank consists of synthesis filters $F_m(z)$, and interpolators with interpolation rate equal to D. The task of the synthesis filter bank is to transform M subband signals $Y_m(z)$ to a full band signal $Y(z)$, which is sampled at the original higher rate, see Fig. (31).

A special uniform filter bank is the modulated filter bank where the individual filters in the filter bank are modulated from prototype filters, $H(z)$ and $F(z)$. Example of such filter bank is the uniform DFT (Discrete Fourier Transform) filter bank. Uniform DFT filter banks may be efficiently implemented using the polyphase implementation employing fast transforms such as the (DFT), and the noble identity properties of decimation and interpolation.

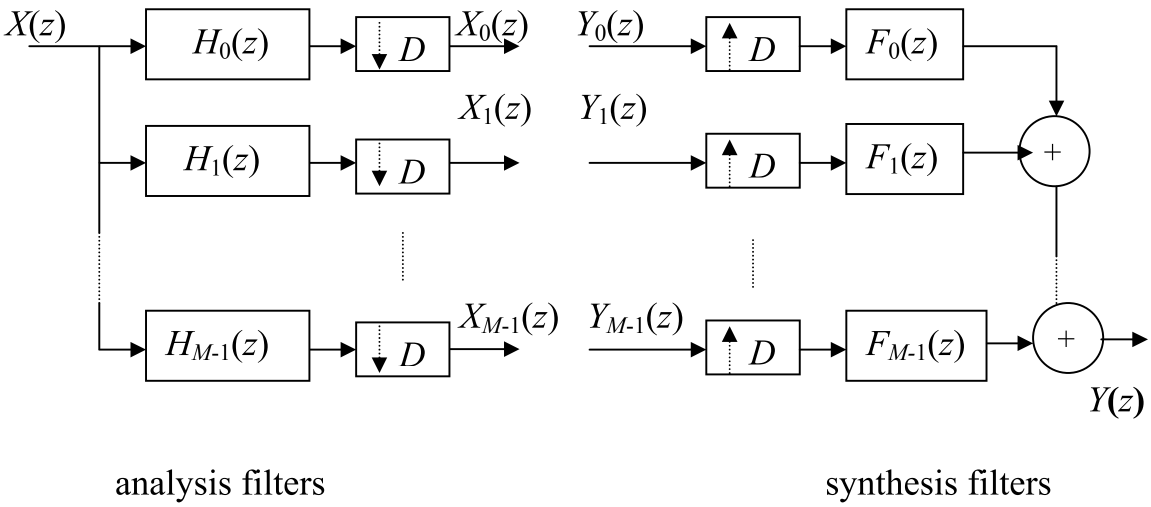


Figure (31)
Uniform analysis and synthesis filter banks.

4.4.2 Uniform DFT Filter Banks

An M-channel Uniform DFT Filter Bank consists of a set of M branches. Each branch consists of an analysis filter $H_m(z)$, a decimator with decimation factor D, an interpolator with interpolation factor D, and a synthesis filter $F_m(z)$. An illustration of such a filter bank is given in Fig. (31),

where subband filtering is applied in the subband signals. All filters have equal bandwidth and the same decimation and interpolation factors which equal to the number of channel, so $M=D$ in this system. Also the length of the filter must equal to the number of channels in order to have efficient polyphase implementation.

In order to construct a uniform filter bank, which means that all filters have the same bandwidth, prototype lowpass analysis filter $H_0(z)$ and prototype lowpass synthesis filter $F_0(z)$ are defined, then all the analysis filters in the filter banks are modulated versions of the prototype lowpass analysis filter $H_0(z)$, and by using polyphase implementation, the synthesis filters is obtained, so we need here just to design a single filter $H_0(z)$, then we obtain the overall filters banks system.

4.5 Design Procedures of Multirate Digital Filter Banks

This section presents the design steps for a uniform DFT filter banks, by using polyphase implementation.

4.5.1 Analysis Filter Bank Design

The analysis filter bank design problem reduces to the design of a single prototype filter $H_0(z)$, when the analysis filters are actually modulated versions of the single prototype analysis filter $H_0(z)$, according to eq. (4.9).

$$H_m(z) = H_0(z.e^{-j2\pi m/M}) \tag{4.9}$$

The purpose of the analysis filters is to split the original signal into a set of subband signals.

Now assume that, the analysis filter group are

$$H_m(z) = H_0(z), H_1(z), H_3(z), \ldots\ldots\ldots\ldots.., H_{M-1}(z) \tag{4.10}$$

with

$$H_m(z) = H_0(z.W_M^m) \tag{4.11}$$

where

$$W_M^m = e^{(-2\pi jm/M)} \tag{4.12}$$

Now by using polyphase implementation (Type I polyphase) as according to eq. (4,6).

$$\begin{aligned} H_m(z) &= H_0(z.e^{-j2\pi m/M}) = \sum_{l=0}^{M-1} z^{-l}.e^{j2\pi ml/M}.E_l(z^M \overbrace{e^{-j2\pi mM/M}}^{1}) \\ &= \sum_{l=0}^{M-1} z^{-l}.W^{-ml}.E_l(z^M) \end{aligned} \tag{4.13}$$

So the overall analysis bank system becomes as follow:

$$\begin{bmatrix} H_0(z) \\ H_1(z) \\ H_2(z) \\ \vdots \\ H_{M-1}(z) \end{bmatrix} = \overbrace{\begin{bmatrix} W^0 & W^0 & W^0 & \dots & W^0 \\ W^0 & W^{-1} & W^{-2} & \dots & W^{-(M-1)} \\ W^0 & W^{-2} & W^{-4} & \dots & W^{-2(M-1)} \\ \vdots & \vdots & \vdots & & \vdots \\ W^0 & W^{-(M-1)} & W^{-2(M-1)} & \dots & W^{-(M-1)^2} \end{bmatrix}}^{F^*} . \begin{bmatrix} E_0(z^M) \\ z^{-1}.E_1(z^M) \\ z^{-2}.E_2(z^M) \\ \vdots \\ z^{-(M-1)}.E_{N-1}(z^M) \end{bmatrix} \tag{4.14}$$

where

$$W = e^{-j2\pi/M}$$

and F is $M * M$ DFT matrix.

Design Example (4.1)

Required to design an FIR uniform analysis filters bank of a uniform filter banks, where the number of channel equal to the decimation factor of 4, the cutoff frequency of lowpass filter equal to $1/M$, the stopband Attenuation ripple equal to the passband attenuation ripple, which is adjusted to give the parameter *ß* equal to 1.

Solution:

From the assumption we find that the filter order of each filter equal to M-1, so the filter order for each filters equal to three.

Now we need to design a single lowpass filter $H_0(z)$ by using Kaiser window, so the transfer function of $H_0(z)$ is

$$H_0(z) = 0.1973 + 0.3027\,z^{-1} + 0.3027\,z^{-2} + 0.1973\,z^{-3}$$

We have here four analysis filters banks, so by using polyphase implementation (Type I) as in eq. (4.14) we find that the transfer functions of the analysis filters are

$$H_0(z) = 0.1973 + 0.3027\, z^{-1} + 0.3027\, z^{-2} + 0.1973\, z^{-3}$$

$$H_1(z) = 0.1973 - (0.1514 - 0.2622j)\, z^{-1} - (0.1514 + 0.2622j)\, z^{-2} + 0.1973\, z^{-3}$$

$$H_2(z) = 0.1973 - (0.1514 + 0.2622j)\, z^{-1} - (0.1514 - 0.2622j)\, z^{-2} + 0.1973\, z^{-3}$$

$$H_3(z) = 0.1973 + 0.3027\, z^{-1} + 0.3027\, z^{-2} + 0.1973\, z^{-3}$$

The prototype filter is FIR low pass filter with order three. The Kaiser window is used to design this filter. Also the other filters are a modulated version of the prototype filter, some of these filters are complex FIR filters which means, some of the filter coefficients are complex number. We see that the length of each analysis filters equal to the number of channel, so that the analysis filters order equal to three for each analysis filter in this example.

The magnitude response shown in Fig. (32), proves that the magnitude response of the analysis filters bank are uniformly shifted version of the response of the basic filter $H_0(z)$ and all filters has the same transition width. The phase response of these filters system are linear as shown in Fig. (33) since there are a symmetry in the coefficients of these filters, therefore the analysis uniform FIR filter banks are linear.

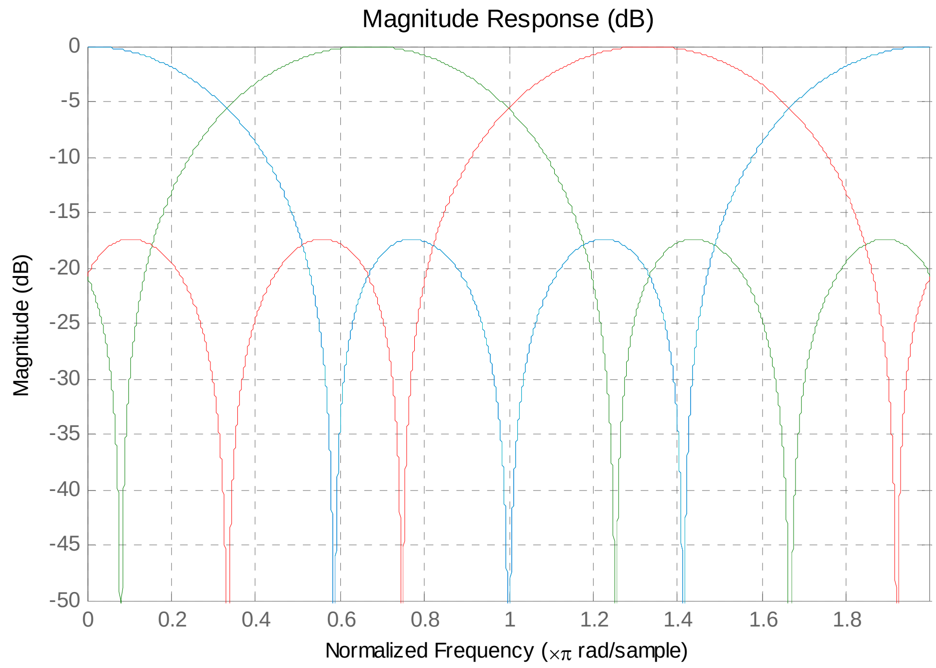


Figure (32)
The magnitude response of the FIR analysis filters bank for *M*=4.

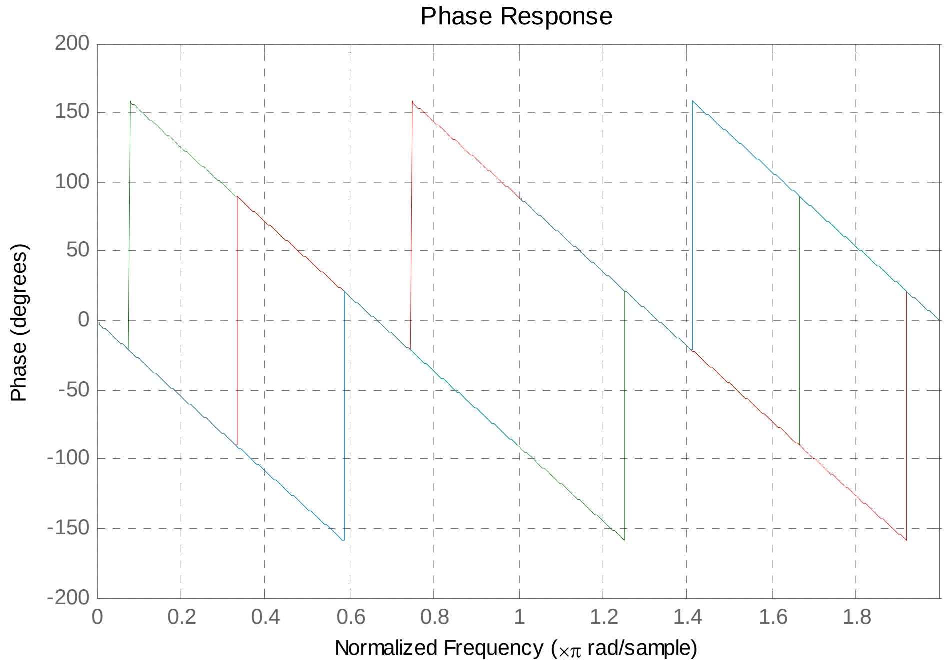


Figure (33)
The phase response of the FIR analysis filters bank for *M*=4.

Design Example (4.2)

Require to design an FIR analysis filters bank of a uniform filter banks, with the same specification such as in example (4.1) and the number of channel equal to 10, and the cutoff frequency of lowpass filter equal to $1/M$.

Solution:

We have here ten analysis filters with filter order nine for each filter, so the length of each filter equal to ten.

Kaiser window is used to design the prototype filter $H_0(z)$, also some of the analysis filters have a complex coefficients value.

The transfer functions of the analysis filters banks are

$$H_0(z) = 0.06848 + 0.08747\,z^{-1} + 0.1042\,z^{-2} + 0.1166\,z^{-3} + 0.1233\,z^{-4} + 0.1233\,z^{-5} + 0.1166\,z^{-6} + 0.1042\,z^{-7} + 0.08747\,z^{-8} + 0.06848\,z^{-9}.$$

$$H_1(z) = 0.06848 + (0.06701 + 0.05623j)\,z^{-1} + (0.01809 + 0.1026j)\,z^{-2} - (0.0583 - 0.101j)\,z^{-3} - (0.1158 - 0.04216j)\,z^{-4} - (0.1158 + 0.04216j)\,z^{-5} - (0.05831 + 0.101j)\,z^{-6} + (0.01809 - 0.1026j)\,z^{-7} + (0.06701 - 0.05623j)z^{-8} + 0.06848\,z^{-9}.$$

$$H_2(z) = 0.06848 + (0.01519 + 0.08614j)\,z^{-1} - (0.09789 - 0.03563j)\,z^{-2} - (0.05831 + 0.101j)\,z^{-3} + (0.09442 - 0.07923j)\,z^{-4} + (0.09442 + 0.07923j)\,z^{-5} - (0.05831 - 0.101j)\,z^{-6} - (0.09789 + 0.03563j)\,z^{-7} + (0.01519 - 0.08614j)\,z^{-8} + 0.06848\,z^{-9}.$$

$$H_3(z) = 0.06848 - (0.04374 - 0.07575j)\,z^{-1} - (0.05209 + 0.09022j)\,z^{-2} + (0.1166)\,z^{-3} - (0.06163 - 0.1067j)\,z^{-4} - (0.06163 + 0.1067j)\,z^{-5} + (0.1166)\,z^{-6} - (0.05209 - 0.09022j)\,z^{-7} - (0.04374 + 0.07575j)\,z^{-8} + 0.06848\,z^{-9}.$$

$$H_4(z) = 0.06848 - (0.0822 - 0.02992j)\,z^{-1} + (0.0798 - 0.06696j)\,z^{-2} - (0.05831 - 0.101j)\,z^{-3} + (0.0214 - 0.1214j)\,z^{-4} + (0.0214 + 0.1214j)\,z^{-5} - (0.05831 + 0.101j)\,z^{-6} + (0.0798 + 0.06696j)\,z^{-7} - (0.0822 + 0.02992j)\,z^{-8} + 0.06848\,z\text{^-9}.$$

$$H_5(z) = 0.06848 - (0.0822 + 0.02992j)\,z^{-1} + (0.0798 + 0.06696j)\,z^{-2} - (0.05831 + 0.101j)\,z^{-3} + (0.0214 + 0.1214j)\,z^{-4} + (0.0214 - 0.1214j)\,z^{-5} - (0.05831 - 0.101j)\,z^{-6} + (0.0798 - 0.06696j)\,z^{-7} - (0.0822 - 0.02992j)\,z^{-8} + 0.06848\,z^{-9}.$$

$$H_6(z) = 0.06848 - (0.04374 + 0.07575j)\,z^{-1} - (0.05209 - 0.09022j)\,z^{-2} + (0.1166)\,z^{-3} - (0.06163 + 0.1067j)\,z^{-4} - (0.06163 - 0.1067j)\,z^{-5} + (0.1166)\,z^{-6} - (0.05209 + 0.09022j)\,z^{-7} - (0.04374 - 0.07575j)\,z^{-8} + 0.06848\,z^{-9}.$$

$$H_7(z) = 0.06848 + (0.01519 - 0.08614j)\,z^{-1} - (0.09789 + 0.03563j)\,z^{-2} - (0.05831 - 0.101j)\,z^{-3} + (0.09442 + 0.07923j)\,z^{-4} + (0.09442 - 0.07923j)\,z^{-5} - (0.05831 + 0.101j)\,z^{-6} - (0.09789 - 0.03563j)z^{-7} + (0.01519 + 0.08614j)z^{-8} + 0.06848\,z^{-9}.$$

$$H_8(z) = 0.06848 + (0.06701 - 0.05623j)\,z^{-1} + (0.01809 - 0.1026j)\,z^{-2} - (0.05831 + 0.101j)\,z^{-3} - (0.1158 + 0.04216j)\,z^{-4} - (0.1158 - 0.04216j)\,z^{-5} - (0.05831 - 0.101j)\,z^{-6} + (0.01809 + 0.1026j)\,z^{-7} + (0.06701 + 0.05623j)z^{-8} + 0.06848\,z^{-9}.$$

$$H_9(z) = 0.06848 + 0.08747\,z^{-1} + 0.1042\,z^{-2} + 0.1166\,z^{-3} + 0.1233\,z^{-4} + 0.1233\,z^{-5} + 0.1166\,z^{-6} + 0.1042\,z^{-7} + 0.08747\,z^{-8} + 0.06848\,z^{-9}.$$

The magnitude response of the analysis filters bank for M=10 is shown in Fig. (34), as we see here that filters have the same bandwidth. While the phase response of the analysis filters bank for M=10 is shown in Fig. (35) where the phase response is also linear, since the filters coefficients are symmetric as given in the transfer function of these filters.

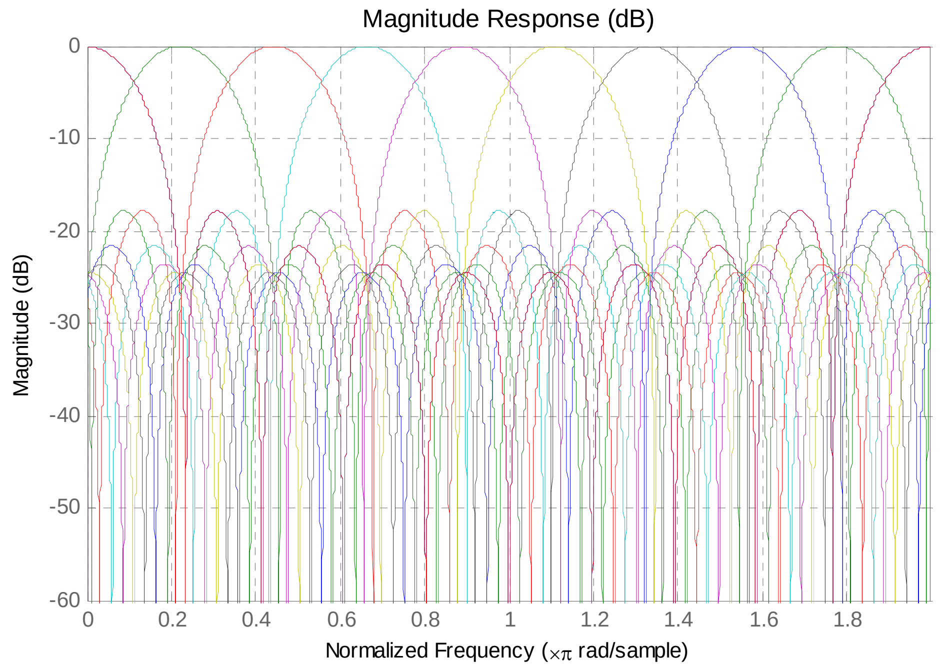


Figure (34)
The magnitude response of the FIR analysis filters bank for *M*=10.

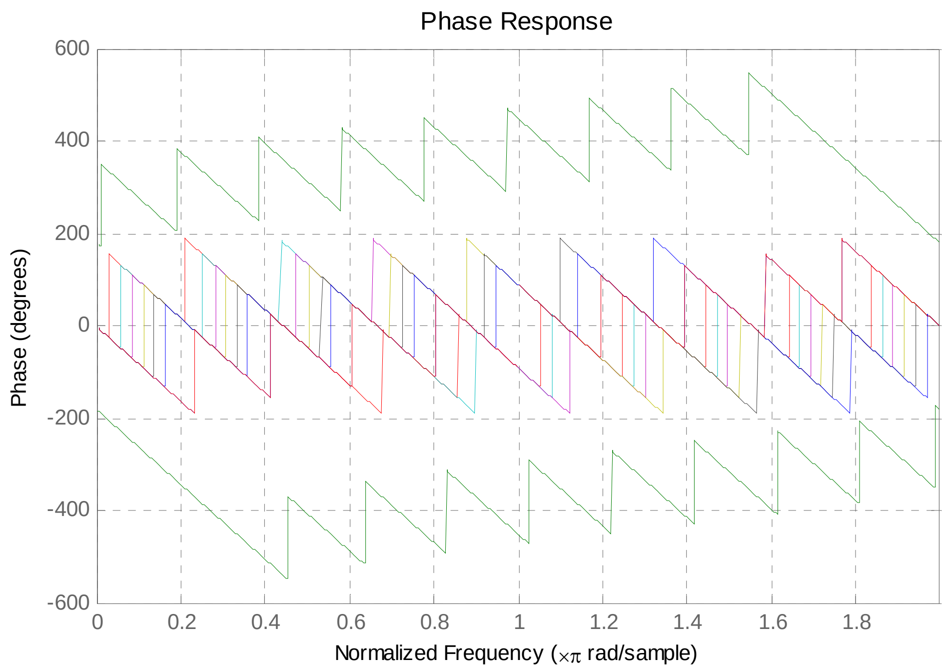


Figure (35)
The phase response of the FIR analysis filters bank for *M*=10.

Design Example(4.3)

Required to design a uniform DFT multirate digital analysis filters bank for IIR elliptic filter with the following specifications:
The passband ripple (δ_p)=0.01, and the stopband ripple (δ_s)=0.001, and the number of channel equal to 4 and the cutoff frequency equal to 1/*M*.

Solution:

We have here four analysis filters with filter order 3 for each filter, so the length of each filter equal to four.

We have here four analysis filters which can obtain from one filter and come as a modulated version of $H_0(z)$, so the transfer function of $H_0(z)$ is

$$H_0(z) = \frac{0.02513 + 0.06047\,z^{-1} + 0.06047\,z^{-2} + 0.02513\,z^{-3}}{1 - 1.865\,z^{-1} + 1.5\,z^{-2} - 0.4641\,z^{-3}}.$$

and by using eq. (4.14) we find the transfer functions of all filters, so the transfer functions of the analysis filters bank are

$$H_0(z) = \frac{0.02513 + 0.06047\,z^{-1} + 0.06047\,z^{-2} + 0.02513\,z^{-3}}{1 - 1.865\,z^{-1} + 1.5\,z^{-2} - 0.4641\,z^{-3}}.$$

$$H_2(z) = \frac{0.02513 - (0.03023 - 0.05237j)\,z^{-1} - (0.03023 + 0.05237j)\,z^{-2} + 0.02513z^{-3}}{1 + (0.9325 - 1.615j)\,z^{-1} - (0.7501 + 1.299j)\,z^{-2} - 0.4641\,z^{-3}}.$$

$$H_2(z) = \frac{0.02513 - (0.03023 + 0.05237j)\,z^{-1} - (0.03023 - 0.05237j)\,z^{-2} + 0.02513\,z^{-3}}{1 + (0.9325 + 1.615j)\,z^{-1} - (0.7501 - 1.299j)\,z^{-2} - 0.4641\,z^{-3}}.$$

$$H_3(z) = \frac{0.02513 + 0.06047\,z^{-1} + 0.06047\,z^{-2} + 0.02513\,z^{-3}}{1 - 1.865\,z^{-1} + 1.5\,z^{-2} - 0.4641\,z^{-3}}.$$

The magnitude response shown in Fig. (36), we see that the magnitude response of the analysis filters bank are uniformly shifted version of the response of the basic filter $H_0(z)$ and all filters has the same transition width.

The phase response of this filters system is non-linear as shown in Fig. (37), since there is a lack of symmetry.

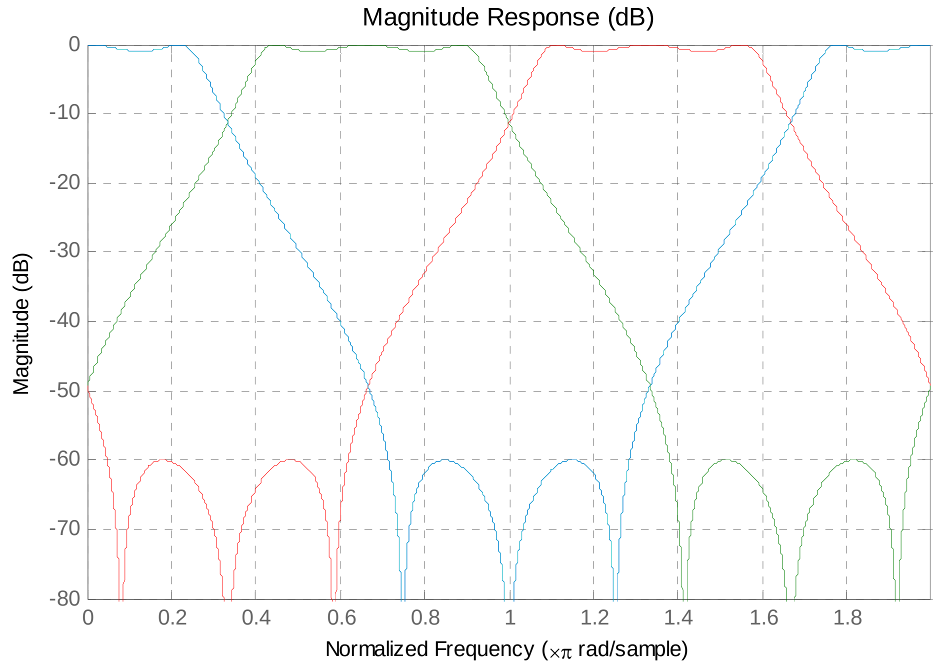


Figure (36)
The magnitude response of the IIR analysis filters bank for *M*=4.

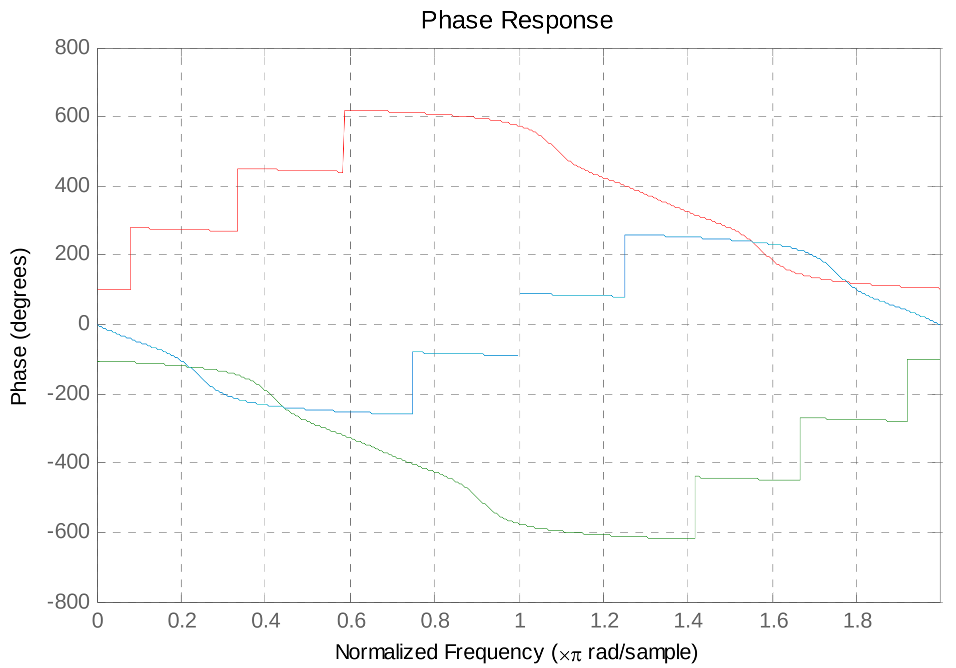


Figure (37)
The phase response of the IIR analysis filters bank for M=4.

4.5.2 Synthesis Filter Bank Design

A corresponding M-channel synthesis filter bank consists of synthesis filters $F_m(z)$, and interpolators with interpolation rate equal to D. The task of the synthesis filter bank is to transform M subband signals to a full band signal, which is sampled at the original higher rate.

Similar to the design of analysis filter banks, presented above, the design of synthesis filter banks reduces to the design of a single prototype filter. When designing the synthesis filter bank, the focus is on the performance of the analysis-synthesis filter bank as a whole. This implies that the synthesis filter bank is designed given an analysis filter bank, given the prototype analysis filter, designed using the method presented in the previous section.

To design the synthesis filter bank, the polyphase implementation is applied (Type II polyphase) according to eq. (4.8).

Now assume that, the synthesis filters group are

$$F_m(z) = F_0(z), F_1(z), F_3(z), \ldots\ldots\ldots\ldots.., F_{M-1}(z) \qquad (4.15)$$

With

$$F_m(z) = F_0(z.W_M^m) \qquad (4.16)$$

Now by using polyphase implementation (Type II polyphase) as according to eq. (4.8).

$$F_0(z) = \sum_{l=0}^{M-1} z^{-l}.R_l(z^M) \qquad (4.17)$$

$$F_m(z) = \sum_{l=0}^{M-1} z^{-l}.W^{m(M-1-l)}.R_l(z^M) \qquad (4.18)$$

So the overall synthesis bank system becomes as follow:

$$\left[F_0(z) \quad F_1(z) \quad F_2(z) \quad \ldots \quad F_{M-1}(z)\right]$$

$$=\left[z^{-N+1}.R_{N-1}(z^N)\ \ \ldots\ \ z^{-2}.R_2(z^N)\ \ z^{-1}.R_1(z^N)\ \ R_0(z^N)\right].\overbrace{\begin{bmatrix} W^0 & W^0 & W^0 & \ldots & W^0 \\ W^0 & W^1 & W^2 & \ldots & W^{(N-1)} \\ W^0 & W^2 & W^4 & \ldots & W^{2(N-1)} \\ \vdots & \vdots & \vdots & & \vdots \\ W^0 & W^{(N-1)} & W^{2(N-1)} & \ldots & W^{(N-1)^2} \end{bmatrix}}^{F} \qquad (4.19)$$

where

$$W = e^{-j2\pi/N}$$

and F is $M * M$ DFT matrix.

For perfect reconstruction system the following condition must be satisfied

$$R(z)E(z)=I \qquad (4.20)$$

where I is the identity matrix. Then the output signal unchanged, and will be as the input signal.

Design Example (4.4)

Required to design a synthesis FIR filters bank of a uniform DFT filter banks, use the same specifications as in example (4.1).

Solution:

We have here four synthesis filters with filter order 3 for each filter, so the length of each filter equal to 4.

We find that the transfer function of $F_0(z)$ is given by

$$F_0(z) = 0.1973 + 0.3027\, z^{-1} + 0.3027\, z^{-2} + 0.1973\, z^{-3}$$

We have here four synthesis filters banks, now by using the polyphase implementation (Type II) as given by eq. (4.19) we find that the transfer functions of the synthesis filters are

$$F_0(z) = 0.1973 + 0.3027\,z^{-1} + 0.3027\,z^{-2} + 0.1973\,z^{-3}$$

$$F_1(z) = 0.1973 - (0.1514 - 0.2622j)\,z^{-1} - (0.1514 + 0.2622j)\,z^{-2} + 0.1973\,z^{-3}$$

$$F_2(z) = 0.1973 - (0.1514 + 0.2622j)\,z^{-1} - (0.1514 - 0.2622j)\,z^{-2} + 0.1973\,z^{-3}$$

$$F_3(z) = 0.1973 + 0.3027\,z^{-1} + 0.3027\,z^{-2} + 0.1973\,z^{-3}$$

Some of these filters are complex FIR, since some of the filter coefficients are complex number.

The magnitude response shown in Fig. (38), can be seen to be uniform, since all filters have the same transition width. Also the phase response of this system is shown in Fig. (39), we have here linear phase FIR synthesis filter banks system.

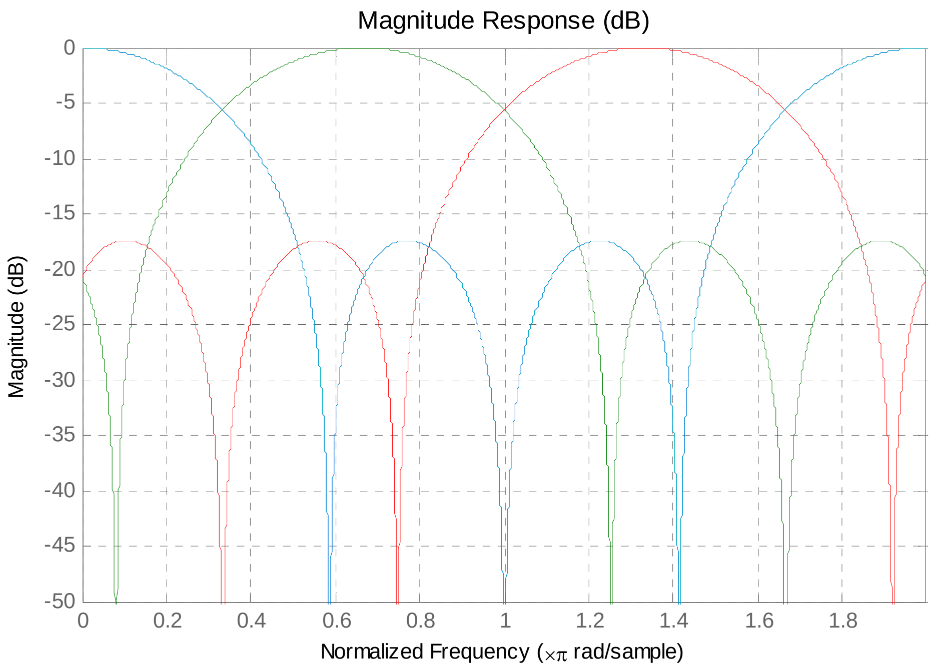


Figure (38)
The magnitude response of the FIR synthesis filters bank for *M*=4.

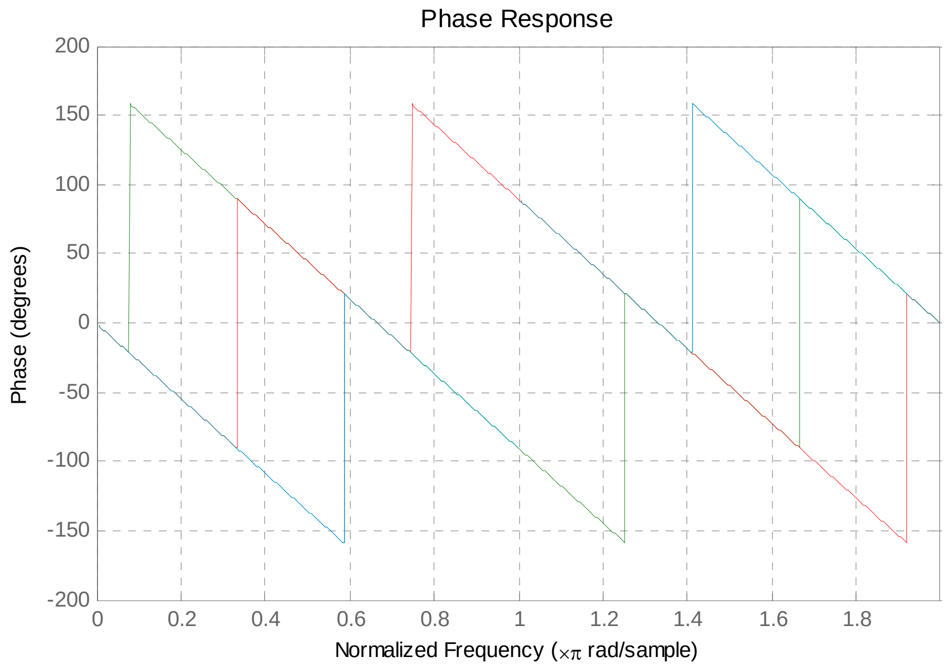


Figure (39)

The phase response of the FIR synthesis filters bank for *M*=4.

Design Example (4.5)

Required to design a synthesis FIR filters bank of a uniform DFT filter banks, where the number of channel equal to the interpolation factor, which equals 10 use the same specifications as in example (4.1).

Solution:

We have here ten synthesis filters with filter order nine for each filter, so the length of each filter equal to ten. Some of these filters are complex FIR, since some of the filter coefficients are complex number.
We find that the transfer function of $F_0(z)$ is given by

$$F_0(z) = 0.06848 + 0.08747\,z^{-1} + 0.1042\,z^{-2} + 0.1166\,z^{-3} + 0.1233\,z^{-4} + 0.1233\,z^{-5} + 0.1166\,z^{-6} + 0.1042\,z^{-7} + 0.08747\,z^{-8} + 0.06848\,z^{-9}.$$

We have here ten synthesis filters banks, now by using the polyphase implementation (Type II) as given by eq.(4.19) we find that the transfer functions of the synthesis filters are

$$F_0(z) = 0.06848 + 0.08747\,z^{-1} + 0.1042\,z^{-2} + 0.1166\,z^{-3} + 0.1233\,z^{-4} + 0.1233\,z^{-5} + 0.1166\,z^{-6} + 0.1042\,z^{-7} + 0.08747\,z^{-8} + 0.06848\,z^{-9}.$$

$$F_1(z) = 0.06848 + (0.06701 + 0.05623j)\,z^{-1} + (0.01809 + 0.1026j)\,z^{-2} - (0.0583 - 0.101j)\,z^{-3} - (0.1158 - 0.04216j)\,z^{-4} - (0.1158 + 0.04216j)\,z^{-5} - (0.05831 + 0.101j)\,z^{-6} + (0.01809 - 0.1026j)\,z^{-7} + (0.06701 - 0.05623j)z^{-8} + 0.06848\,z^{-9}.$$

$$F_2(z) = 0.06848 + (0.01519 + 0.08614j)\,z^{-1} - (0.09789 - 0.03563j)\,z^{-2} - (0.05831 + 0.101j)\,z^{-3} + (0.09442 - 0.07923j)\,z^{-4} + (0.09442 + 0.07923j)\,z^{-5} - (0.05831 - 0.101j)\,z^{-6} - (0.09789 + 0.03563j)\,z^{-7} + (0.01519 - 0.08614j)\,z^{-8} + 0.06848\,z^{-9}.$$

$$F_3(z) = 0.06848 - (0.04374 - 0.07575j)\,z^{-1} - (0.05209 + 0.09022j)\,z^{-2} + (0.1166)\,z^{-3} - (0.06163 - 0.1067j)\,z^{-4} - (0.06163 + 0.1067j)\,z^{-5} + (0.1166)\,z^{-6} - (0.05209 - 0.09022j)\,z^{-7} - (0.04374 + 0.07575j)\,z^{-8} + 0.06848\,z^{-9}.$$

$$F_5(z) = 0.06848 - (0.0822 + 0.02992j)\,z^{-1} + (0.0798 + 0.06696j)\,z^{-2} - (0.05831 + 0.101j)\,z^{-3} + (0.0214 + 0.1214j)\,z^{-4} + (0.0214 - 0.1214j)\,z^{-5} - (0.05831 - 0.101j)\,z^{-6} + (0.0798 - 0.06696j)\,z^{-7} - (0.0822 - 0.02992j)\,z^{-8} + 0.06848\,z^{-9}.$$

$$F_6(z) = 0.06848 - (0.04374 + 0.07575j)\,z^{-1} - (0.05209 - 0.09022j)\,z^{-2} + (0.1166)\,z^{-3} - (0.06163 + 0.1067j)\,z^{-4} - (0.06163 - 0.1067j)\,z^{-5} + (0.1166)\,z^{-6} - (0.05209 + 0.09022j)\,z^{-7} - (0.04374 - 0.07575j)\,z^{-8} + 0.06848\,z^{-9}.$$

$$F_7(z) = 0.06848 + (0.01519 - 0.08614j)\,z^{-1} - (0.09789 + 0.03563j)\,z^{-2} - (0.05831 - 0.101j)\,z^{-3} + (0.09442 + 0.07923j)\,z^{-4} + (0.09442 - 0.07923j)\,z^{-5} - (0.05831 + 0.101j)\,z^{-6} - (0.09789 - 0.03563j)z^{-7} + (0.01519 + 0.08614j)z^{-8} + 0.06848\,z^{-9}.$$

$$F_8(z) = 0.06848 + (0.06701 - 0.05623j)\,z^{-1} + (0.01809 - 0.1026j)\,z^{-2} - (0.05831 + 0.101j)\,z^{-3} - (0.1158 + 0.04216j)\,z^{-4} - (0.1158 - 0.04216j)\,z^{-5} - (0.05831 - 0.101j)\,z^{-6} + (0.01809 + 0.1026j)\,z^{-7} + (0.06701 + 0.05623j)z^{-8} + 0.06848\,z^{-9}.$$

$$F_9(z) = 0.06848 + 0.08747\,z^{-1} + 0.1042\,z^{-2} + 0.1166\,z^{-3} + 0.1233\,z^{-4} + 0.1233\,z^{-5} + 0.1166\,z^{-6} + 0.1042\,z^{-7} + 0.08747\,z^{-8} + 0.06848\,z^{-9}.$$

The magnitude response of the synthesis filters bank for M=10 is shown in Fig. (40), as we see here that each filters have the same bandwidth. While the phase response is shown in Fig. (41) which shows that the synthesis uniform FIR filter banks have a linear phase response.

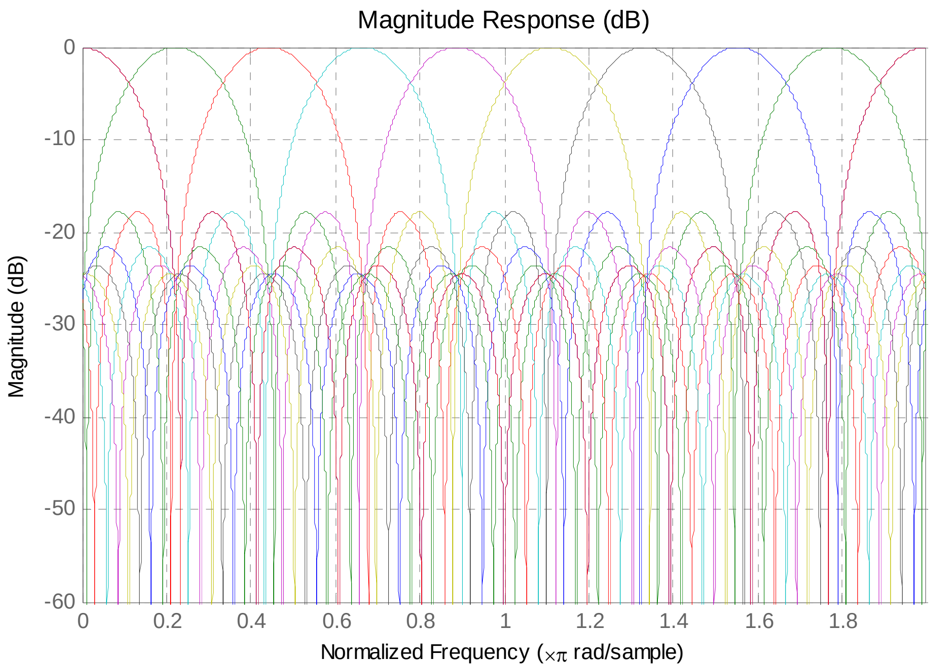


Figure (40)
The magnitude response of the FIR synthesis filters bank for M=10.

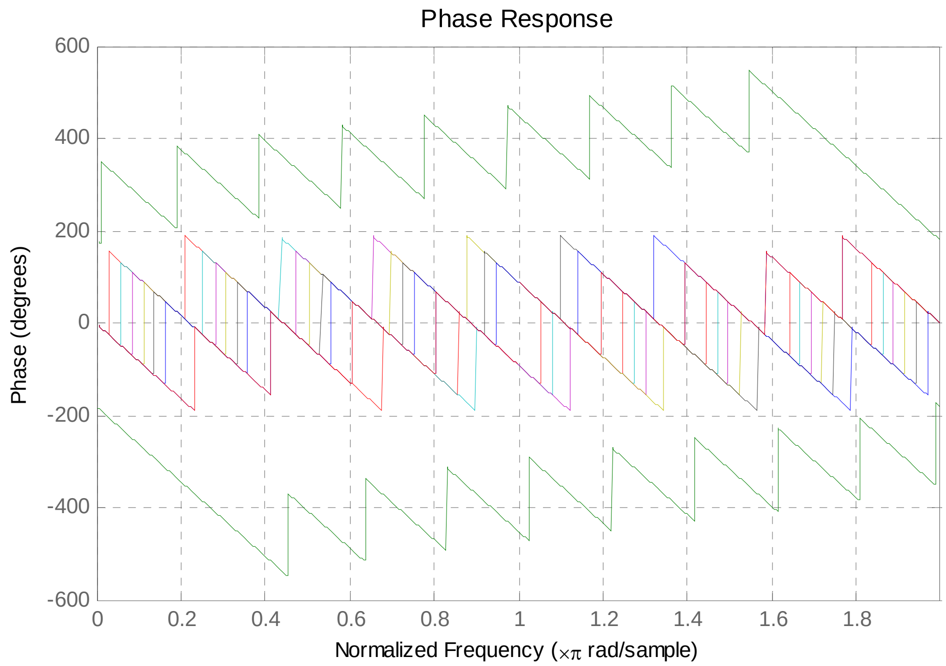


Figure (41)

The phase response of the FIR synthesis filters bank for M=10.

Design Example (4.6)

Required to design a uniform DFT multirate digital synthesis filters bank for IIR elliptic filter with the same specifications such as example (4.3), and the number of channel equal to 4 and the cutoff frequency equal to 1/M.

Solution:

We have here four synthesis filters with filter order three for each filter, so the length of each filter equal to four.

We have here four synthesis filters which can obtain from one filter and come as a modulated version of $H_0(z)$, so the transfer function of $H_0(z)$ is

$$F_0(z) = \frac{0.02513 + 0.06047\,z^{-1} + 0.06047\,z^{-2} + 0.02513\,z^{-3}}{1 - 1.865\,z^{-1} + 1.5\,z^{-2} - 0.4641\,z^{-3}}.$$

We have here four synthesis filters, now by using the polyphase implementation (Type II) as given by eq.(4.19) we find that the transfer functions of the synthesis filters are

$$F_0(z) = \frac{0.02513 + 0.06047\,z^{-1} + 0.06047\,z^{-2} + 0.02513\,z^{-3}}{1 - 1.865\,z^{-1} + 1.5\,z^{-2} - 0.4641\,z^{-3}}.$$

$$F_2(z)=\frac{0.02513-(0.03023-0.05237j)\,z^{-1}-(0.03023+0.05237j)\,z^{-2}+0.02513z^{-3}}{1+(0.9325-1.615j)\,z^{-1}-(0.7501+1.299j)\,z^{-2}-0.4641\,z^{-3}}.$$

$$F_2(z)=\frac{0.02513-(0.03023+0.05237j)\,z^{-1}-(0.03023-0.05237j)\,z^{-2}+0.02513\,z^{-3}}{1+(0.9325+1.615j)\,z^{-1}-(0.7501-1.299j)\,z^{-2}-0.4641\,z^{-3}}.$$

$$F_3(z)=\frac{0.02513+0.06047\,z^{-1}+0.06047\,z^{-2}+0.02513\,z^{-3}}{1-1.865\,z^{-1}+1.5\,z^{-2}-0.4641\,z^{-3}}.$$

The magnitude response of the synthesis filters bank for M=4 is shown in Fig. (42), as we see here that each filters have the same bandwidth. While the phase response is shown in Fig. (43) which shows that the synthesis IIR filter banks have a non-linear phase response.

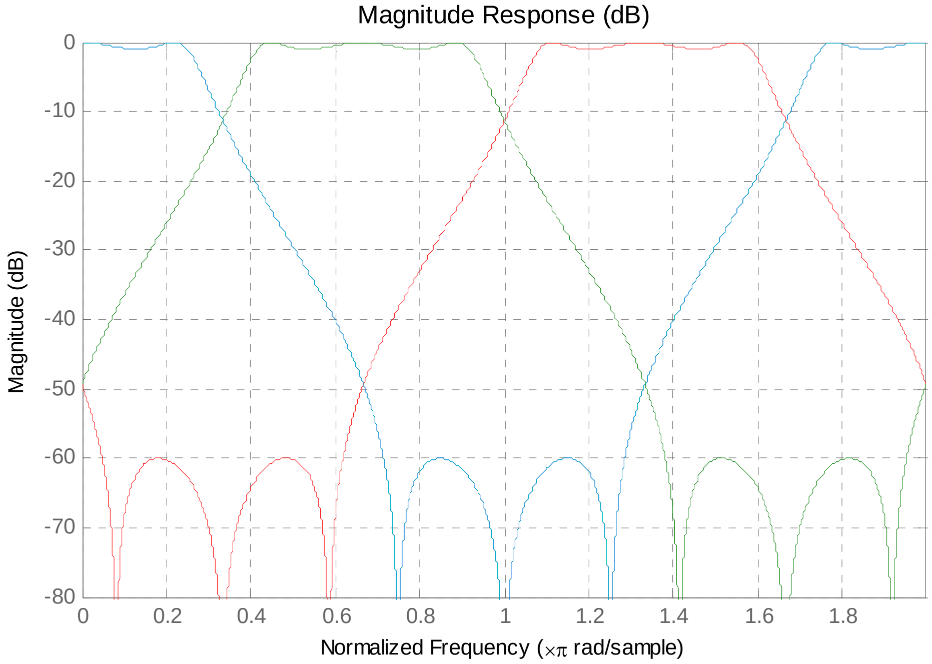


Figure (42)
The magnitude response of the IIR synthesis filters bank for M=4.

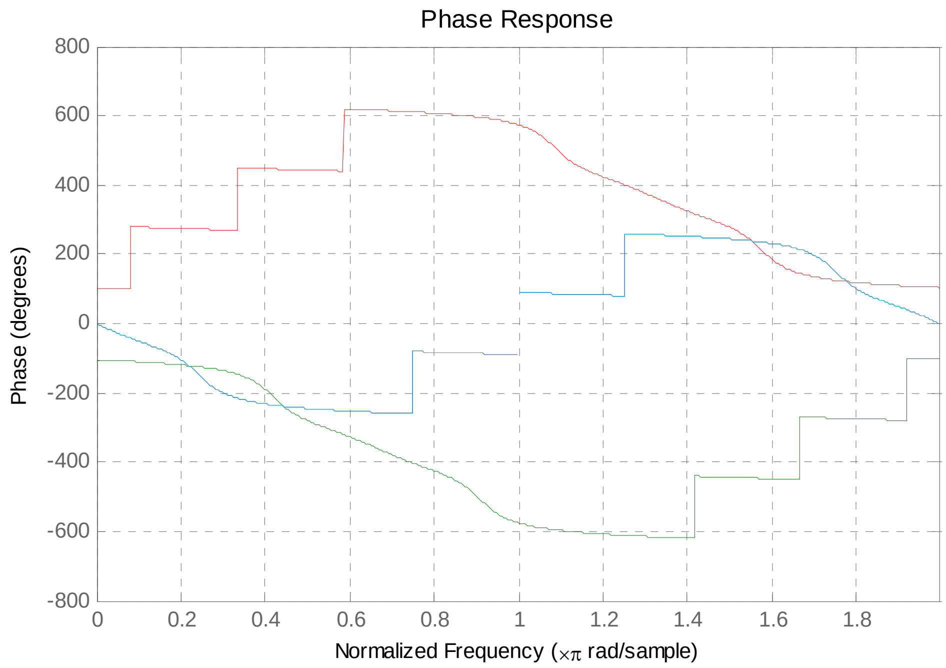


Figure (43)
The phase response of the IIR synthesis filters bank for M=4.

4.5.3 The Overall System (Analysis and Synthesis) Filter Banks Design

In this section I will give a design example to find the analysis and synthesis filters banks for a uniform DFT filter banks using the polyphase implementation.

Design Example (4.7)

Design an analysis and synthesis FIR filters bank of a uniform DFT filter banks, where the number of channels are 5, use the same specifications as in example (4.1).

Solution:

We have here five analysis ,and five synthesis filters, with filter order four for each filters in this system.

By using polyphase representation (Type I) as in eq. (4.14) we find that the transfer functions of the analysis filters banks are

$$H_0(z) = 0.1508 + 0.223\, z^{-1} + 0.2523\, z^{-2} + 0.223\, z^{-3} + 0.1508\, z^{-4}.$$

$$H_1(z) = 0.1508 + (0.223j)\, z^{-1} - (0.2523)\, z^{-2} - (0.223j)\, z^{-3} + 0.1508\, z^{-4}.$$

$$H_2(z) = 0.1508 - (0.223)\,z^{-1} + (0.2523)\,z^{-2} - (0.223)\,z^{-3} + 0.1508\,z^{-4}.$$

$$H_3(z) = 0.1508 - (0.223j)\,z^{-1} - (0.2523)\,z^{-2} + (0.223j)\,z^{-3} + 0.1508\,z^{-4}.$$

$$H_4(z) = 0.1508 + 0.223\,z^{-1} + 0.2523\,z^{-2} + 0.223\,z^{-3} + 0.1508\,z^{-4}.$$

By using polyphase representation (Type II) as in eq. (4.19) we find that the transfer functions of the synthesis filters banks are

$$F_0(z) = 0.1508 + 0.223\,z^{-1} + 0.2523\,z^{-2} + 0.223\,z^{-3} + 0.1508\,z^{-4}.$$

$$F_1(z) = 0.1508 + (0.223j)\,z^{-1} - (0.2523)\,z^{-2} - (0.223j)\,z^{-3} + 0.1508\,z^{-4}.$$

$$F_2(z) = 0.1508 - (0.223)\,z^{-1} + (0.2523)\,z^{-2} - (0.223)\,z^{-3} + 0.1508\,z^{-4}.$$

$$F_3(z) = 0.1508 - (0.223j)\,z^{-1} - (0.2523)\,z^{-2} + (0.223j)\,z^{-3} + 0.1508\,z^{-4}.$$

$$F_4(z) = 0.1508 + 0.223\,z^{-1} + 0.2523\,z^{-2} + 0.223\,z^{-3} + 0.1508\,z^{-4}.$$

The magnitude response of the analysis and synthesis filters banks for M=5 are shown in Fig. (44) and (45) respectively.

We see from Fig. (44) and (45), that the magnitude response the analysis and synthesis filters banks, are the same, which means that, the perfect reconstruction of the input signal will be obtained at the output of the synthesis filters banks. Note that the analysis and synthesis transfer function for a particular filter is the same, this result will help the designer in the design process and will reduce the cost and the complexity of the design.

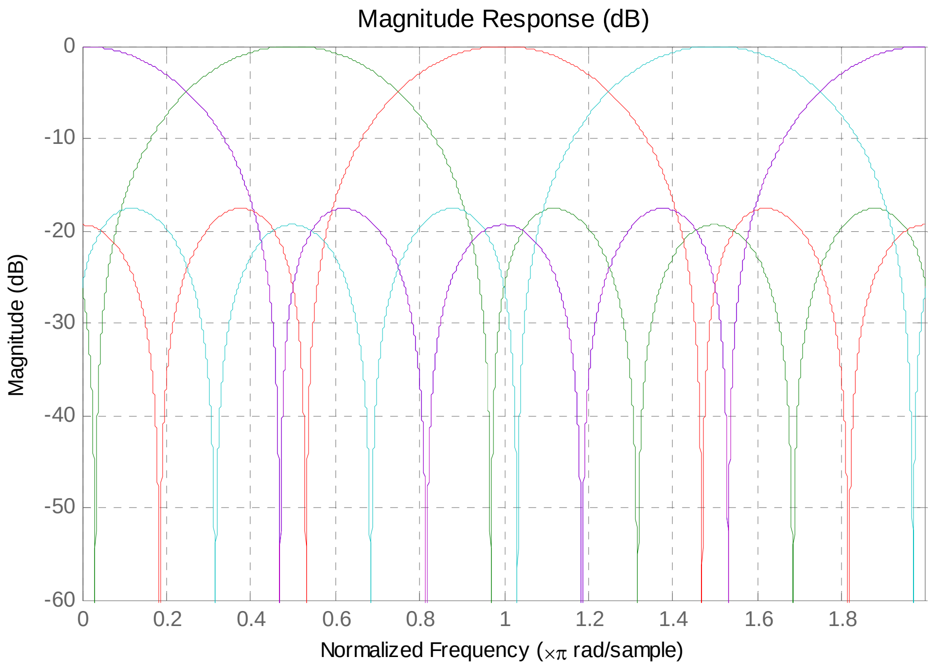


Figure (44)
The magnitude response of the FIR analysis filters bank for *M*=5.

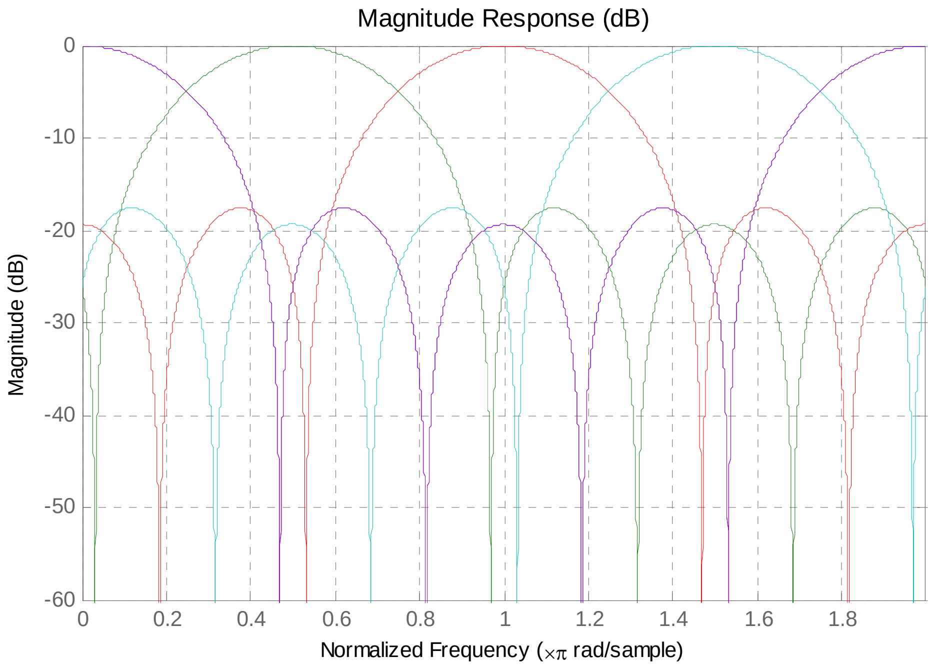


Figure (45)
The magnitude response of the FIR synthesis filters bank for *M*=5.

Design Example (4.8)

Design an analysis and synthesis FIR filters bank of a uniform DFT filter banks, where the number of channels are 8, use the same specifications as in example (4.1).

Solution:

We have here eight analysis, and eight synthesis filters, with filter order seven for each filters in this system.

By using polyphase representation (Type I) as in eq. (4.14) we find that the transfer functions of the analysis filters banks are

$$H_0(z) = 0.08776 + 0.1173\,z^{-1} + 0.1409\,z^{-2} + 0.154\,z^{-3} + 0.154\,z^{-4} + 0.1409\,z^{-5} + 0.1173\,z^{-6} + 0.08776\,z^{-7}.$$

$$H_1(z) = 0.08776 + (0.07314 + 0.09172j)\,z^{-1} - (0.03136 - 0.1374j)\,z^{-2} - (0.1388 - 0.06682j)\,z^{-3} - (0.1388 + 0.06682j)\,z^{-4} - (0.03136 + 0.1374j)\,z^{-5} + (0.07314 - 0.09172j)\,z^{-6} + 0.08776\,z^{-7}.$$

$$H_2(z) = 0.08776 - (0.0261 - 0.1144j)\,z^{-1} - (0.127 + 0.06114j)\,z^{-2} + (0.09602 - 0.1204j)\,z^{-3} + (0.09602 + 0.1204j)\,z^{-4} - (0.127 - 0.06114j)\,z^{-5} - (0.0261 + 0.1144j)\,z^{-6} + 0.08776\,z^{-7}.$$

$$H_3(z) = 0.08776 - (0.1057 - 0.0509j)\,z^{-1} + (0.08786 - 0.1102j)\,z^{-2} - (0.03427 - 0.1502j)\,z^{-3} - (0.03427 + 0.1502j)\,z^{-4} + (0.08786 + 0.1102j)\,z^{-5} - (0.1057 + 0.0509j)\,z^{-6} + 0.08776\,z^{-7}.$$

$$H_4(z) = 0.08776 - (0.1057 + 0.0509j)\,z^{-1} + (0.08786 + 0.1102j)\,z^{-2} - (0.03427 + 0.1502j)\,z^{-3} - (0.03427 - 0.1502j)\,z^{-4} + (0.08786 - 0.1102j)\,z^{-5} - (0.1057 - 0.0509j)\,z^{-6} + 0.08776\,z^{-7}.$$

$$H_5(z) = 0.08776 - (0.0261 + 0.1144j)\,z^{-1} - (0.127 - 0.06114j)\,z^{-2} + (0.09602 + 0.1204j)\,z^{-3} + (0.09602 - 0.1204j)\,z^{-4} - (0.127 + 0.06114j)\,z^{-5} - (0.0261 - 0.1144j)\,z^{-6} + 0.08776\,z^{-7}.$$

$$H_6(z) = 0.08776 + (0.07314 - 0.09172j)\,z^{-1} - (0.03136 + 0.1374j)\,z^{-2} - (0.1388 + 0.06682j)\,z^{-3} - (0.1388 - 0.06682j)\,z^{-4} - (0.03136 - 0.1374j)\,z^{-5} + (0.07314 + 0.09172j)\,z^{-6} + 0.08776\,z^{-7}.$$

$$H_7(z) = 0.08776 + 0.1173\,z^{-1} + 0.1409\,z^{-2} + 0.154\,z^{-3} + 0.154\,z^{-4} + 0.1409\,z^{-5} + 0.1173\,z^{-6} + 0.08776\,z^{-7}.$$

By using polyphase representation (Type II) as in eq. (4.19) we find that the transfer functions of the synthesis filters banks are

$$F_0(z) = 0.08776 + 0.1173\,z^{-1} + 0.1409\,z^{-2} + 0.154\,z^{-3} + 0.154\,z^{-4} + 0.1409\,z^{-5} + 0.1173\,z^{-6} + 0.08776\,z^{-7}.$$

$$F_1(z) = 0.08776 + (0.07314 + 0.09172j)\,z^{-1} - (0.03136 - 0.1374j)\,z^{-2} - (0.1388 - 0.06682j)\,z^{-3} - (0.1388 + 0.06682j)\,z^{-4} - (0.03136 + 0.1374j)\,z^{-5} + (0.07314 - 0.09172j)\,z^{-6} + 0.08776\,z^{-7}.$$

$$F_2(z) = 0.08776 - (0.0261 - 0.1144j)\,z^{-1} - (0.127 + 0.06114j)\,z^{-2} + (0.09602 - 0.1204j)\,z^{-3} + (0.09602 + 0.1204j)\,z^{-4} - (0.127 - 0.06114j)\,z^{-5} - (0.0261 + 0.1144j)\,z^{-6} + 0.08776\,z^{-7}.$$

$$F_3(z) = 0.08776 - (0.1057 - 0.0509j)\,z^{-1} + (0.08786 - 0.1102j)\,z^{-2} - (0.03427 - 0.1502j)\,z^{-3} - (0.03427 + 0.1502j)\,z^{-4} + (0.08786 + 0.1102j)\,z^{-5} - (0.1057 + 0.0509j)\,z^{-6} + 0.08776\,z^{-7}.$$

$$F_4(z) = 0.08776 - (0.1057 + 0.0509j)\,z^{-1} + (0.08786 + 0.1102j)\,z^{-2} - (0.03427 + 0.1502j)\,z^{-3} - (0.03427 - 0.1502j)\,z^{-4} + (0.08786 - 0.1102j)\,z^{-5} - (0.1057 - 0.0509j)\,z^{-6} + 0.08776\,z^{-7}.$$

$$F_5(z) = 0.08776 - (0.0261 + 0.1144j)\,z^{-1} - (0.127 - 0.06114j)\,z^{-2} + (0.09602 + 0.1204j)\,z^{-3} + (0.09602 - 0.1204j)\,z^{-4} - (0.127 + 0.06114j)\,z^{-5} - (0.0261 - 0.1144j)\,z^{-6} + 0.08776\,z^{-7}.$$

$$F_6(z) = 0.08776 + (0.07314 - 0.09172j)\,z^{-1} - (0.03136 + 0.1374j)\,z^{-2} - (0.1388 + 0.06682j)\,z^{-3} - (0.1388 - 0.06682j)\,z^{-4} - (0.03136 - 0.1374j)\,z^{-5} + (0.07314 + 0.09172j)\,z^{-6} + 0.08776\,z^{-7}.$$

$$F_7(z) = 0.08776 + 0.1173\, z^{-1} + 0.1409\, z^{-2} + 0.154\, z^{-3} + 0.154\, z^{-4} + 0.1409\, z^{-5} + 0.1173\, z^{-6} + 0.08776\, z^{-7}.$$

From Fig. (46) and (47), we see that, each filters have the same bandwidth, which proves that the system is uniform filters banks.

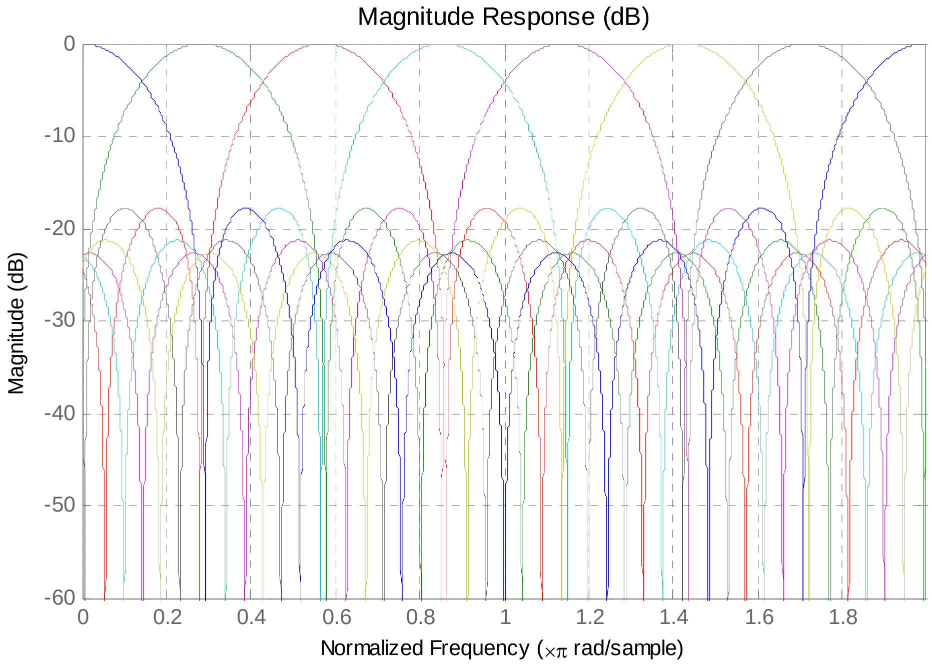


Figure (46)
The magnitude response of the analysis FIR filters banks with *M*=8.

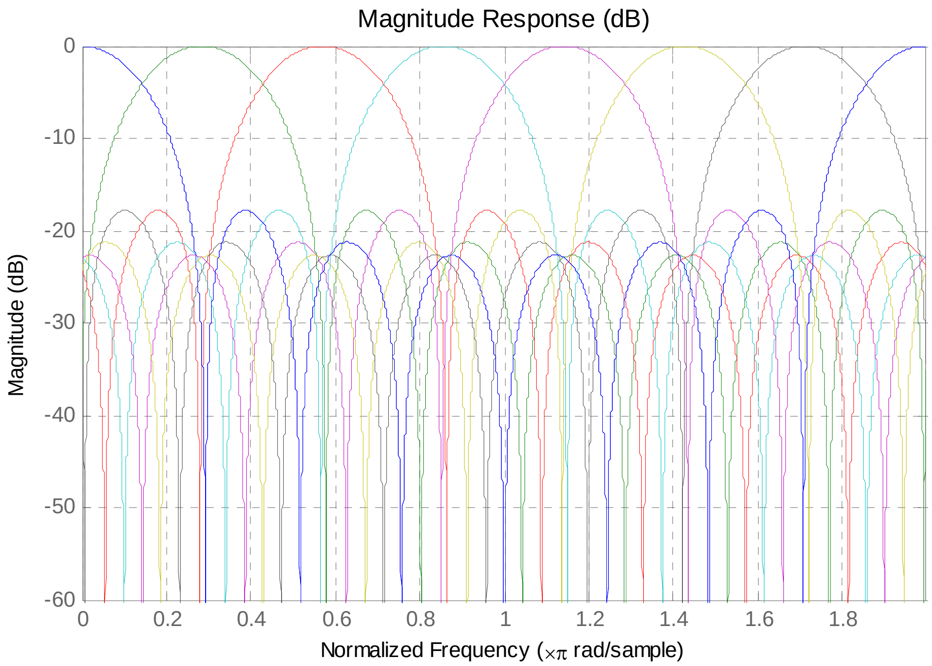


Figure (47)

The magnitude response of the synthesis FIR filters banks with M=8.

Design Example (4.9)

Required to design a uniform DFT multirate digital filters bank for IIR elliptic filter with the same specifications such as example (4.3), and the number of channel equal to 5 and the cutoff frequency equal to $1/M$.

Solution:

We have here five analysis ,and five synthesis filters, with filter order four for each filters in this system.

By using polyphase representation (Type I) as in eq. (4.14) we find that the transfer functions of the analysis filters banks are

$$H_0(z) = \frac{0.005879 + 0.005337z^{-1} + 0.009632z^{-2} + 0.005337z^{-3} + 0.005879z^{-4}.}{1 - 3.048\,z^{-1} + 3.824\,z^{-2} - 2.293\,z^{-3} + 0.5523\,z^{-4}}$$

$$H_1(z) = \frac{0.005879 + (0.005337i)\,z^{-1} - (0.009632)\,z^{-2} - (0.005337i)\,z^{-3} + 0.005879\,z^{-4}.}{1 - (3.048i)\,z^{-1} - (3.824)\,z^{-2} + (2.293i)\,z^{-3} + 0.5523\,z^{-4}}$$

$$H_2(z) = \frac{0.005879 - (0.005337)\,z^{-1} + (0.009632)\,z^{-2} - (0.005337)\,z^{-3} + 0.005879\,z^{-4}.}{1 + (3.048)\,z^{-1} + (3.824)\,z^{-2} + (2.293)\,z^{-3} + 0.5523\,z^{-4}}$$

$$H_3(z) = \frac{0.005879 - (0.005337i)\,z^{-1} - (0.009632)\,z^{-2} + (0.005337i)\,z^{-3} + 0.005879\,z^{-4}}{1 + (3.048i)\,z^{-1} - (3.824)\,z^{-2} - (2.293i)\,z^{-3} + 0.5523\,z^{-4}}.$$

$$H_4(z) = \frac{0.005879 + 0.005337z^{-1} + 0.009632z^{-2} + 0.005337z^{-3} + 0.005879z^{-4}}{1 - 3.048\,z^{-1} + 3.824\,z^{-2} - 2.293\,z^{-3} + 0.5523\,z^{-4}}.$$

By using polyphase representation (Type II) as in eq. (4.19) we find that the transfer functions of the synthesis filters banks are

$$F_0(z) = \frac{0.005879 + 0.005337z^{-1} + 0.009632z^{-2} + 0.005337z^{-3} + 0.005879z^{-4}}{1 - 3.048\,z^{-1} + 3.824\,z^{-2} - 2.293\,z^{-3} + 0.5523\,z^{-4}}.$$

$$F_1(z) = \frac{0.005879 + (0.005337i)\,z^{-1} - (0.009632)\,z^{-2} - (0.005337i)\,z^{-3} + 0.005879\,z^{-4}}{1 - (3.048i)\,z^{-1} - (3.824)\,z^{-2} + (2.293i)\,z^{-3} + 0.5523\,z^{-4}}.$$

$$F_2(z) = \frac{0.005879 - (0.005337)\,z^{-1} + (0.009632)\,z^{-2} - (0.005337)\,z^{-3} + 0.005879\,z^{-4}}{1 + (3.048)\,z^{-1} + (3.824)\,z^{-2} + (2.293)\,z^{-3} + 0.5523\,z^{-4}}.$$

$$F_3(z) = \frac{0.005879 - (0.005337i)\,z^{-1} - (0.009632)\,z^{-2} + (0.005337i)\,z^{-3} + 0.005879\,z^{-4}}{1 + (3.048i)\,z^{-1} - (3.824)\,z^{-2} - (2.293i)\,z^{-3} + 0.5523\,z^{-4}}.$$

$$F_4(z) = \frac{0.005879 + 0.005337z^{-1} + 0.009632z^{-2} + 0.005337z^{-3} + 0.005879z^{-4}}{1 - 3.048\,z^{-1} + 3.824\,z^{-2} - 2.293\,z^{-3} + 0.5523\,z^{-4}}.$$

The magnitude response of the analysis and synthesis filters banks for M=5 are shown in Fig. (48) and (49) respectively.

We see from Fig. (48) and (49), that the magnitude response the analysis and synthesis filters banks, are the same, which means that, the

perfect reconstruction of the input signal will be obtained at the output of the synthesis filters banks. Also the analysis and synthesis transfer function for a particular filter is the same, this result will help the designer in the design process and will reduce the cost and the complexity of the design.

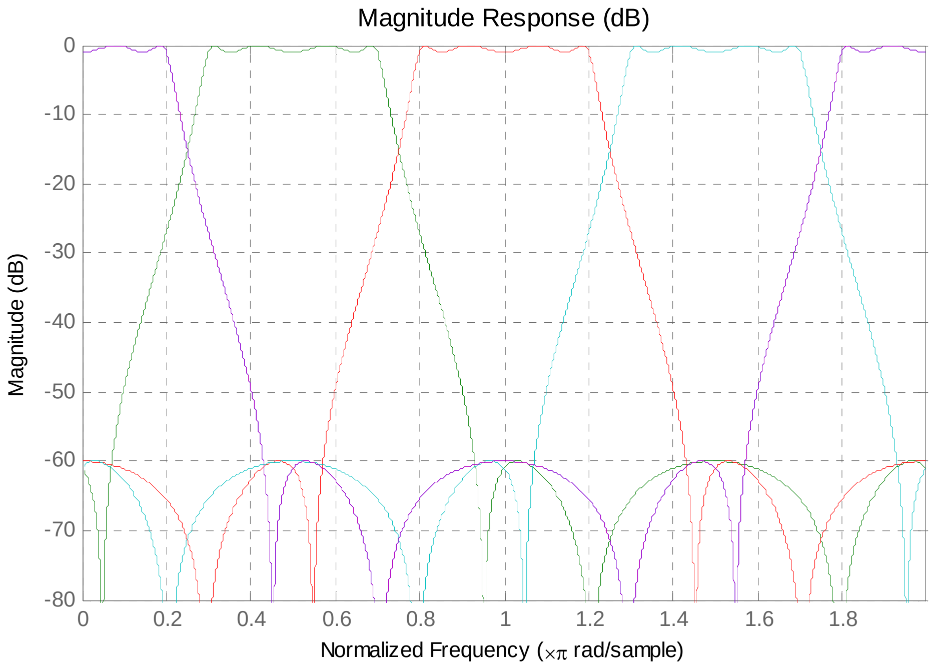


Figure (48)
The magnitude response of the analysis IIR filters banks with M=5.

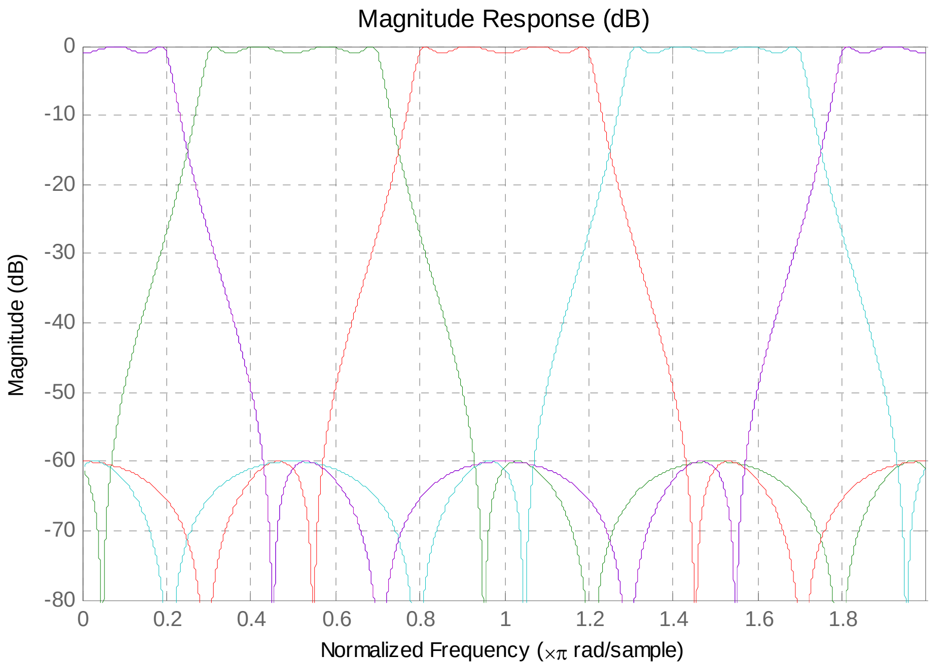


Figure (49)

The magnitude response of the synthesis IIR filters banks with M=5.

4.6 Conclusions

The purpose of the analysis filters banks is to split the original signal into a set of subband signals. While the purpose of the synthesis filters banks is to transform the subband signals to a full band signal, which is sampled at the original higher rate.

To design a uniform DFT filters banks, we need just to design a single prototype lowpass filter, and the other filters are a modulated versions of the lowpass prototype filter.

Polyphase implementation method used to design a multirate digital filters banks is one of great several methods used in this field.

Modulated filter banks provide a computationally efficient implementation, due to the polyphase implementation, and great design simplicity.

To satisfy a perfect reconstruction property of the input signal, the following condition must be satisfied

$$R(z)E(z)=I$$

where I is the identity matrix.

Then the output signal is unchanged, and will be similar to the input signal.

In this thesis, the number of channel must be equal to the decimation factor, the interpolation factor, and the length of the prototype lowpass analysis filter, which equal to the length of all filters in the filters banks systems, in order to obtain efficient polyphase implementation.

Uniform DFT filter banks is a filter bank system in which all filters have the same transition width in both side (analysis and synthesis side). And in this thesis, it was found that the response of the analysis filters banks are the same as that for the synthesis filters banks, which satisfied a perfect reconstruction property. Also the analysis and synthesis transfer function for a particular filter is the same, this result will help the designer in the design process and will reduce the cost and the complexity of the filter banks design system.

It found was that FIR uniform DFT filter banks have a linear phase response, since the coefficients of the filters are symmetric, on the other hand IIR uniform DFT filter banks have a non-linear phase response, due to a lack of filter coefficients symmetry.

Also some of the filters are a complex filter, since some of these filters coefficients are complex value.

Chapter Five
Applications of Mulitirate Digital Filter Banks in Communications

5.1 Introduction

Digital signal processing is an area of science and engineering that has been developed rapidly. Digital signal processing has played a key role in the development of communication systems over the last two decades [26]. The role of digital signal processing in communication systems has been quite significant [27,28,29].

The filter bank has found wide applications in speech and image compression, digital communications, statistical and adaptive signal processing, and many other fields.

Multirate digital filter banks are fundamental systems commonly used in digital signal processing. Multirate digital filter banks have several applications in communication systems, such as transmultiplexer, digital audio systems, subband coding of speech and image signals, antenna systems, and others [30,31,32,33,34,35]. Filter banks also have been used in digital communication systems in many forms. Some of these include filter bank precoding for channel equalization [36,37], equalization with fractionally spaced sampling [38], and discrete multitone modulation [39].

Transmultiplexer is one of the most important applications of the multirate digital filter banks. Transmultiplexer is a system that can be used to convert the time division multiplexing (TDM) signal into a frequency division multiplexing (FDM) signal, this system will be discussed in this chapter as an example of many applications of multirate digital filter banks.

5.2 Transmultiplexer

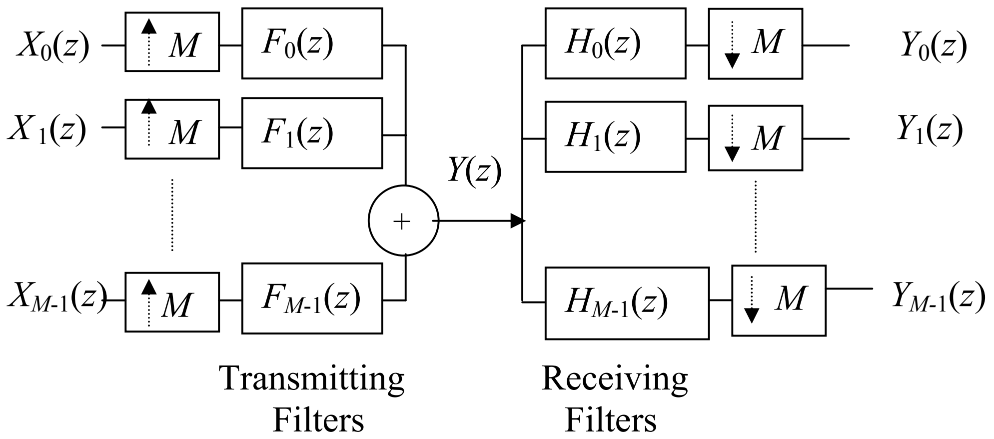
When two or more different signals are composed into a single signal, then a uniform synthesis-analysis system is used, as shown in Fig. (50). This system is also called a transmultiplexer. In this system all the M signals are interpolated by a factor of M and filtered by M synthesis filters $F_k(\mathrm{z})$ for k = 0,1,…, M-1. Then the outputs are added to give a single signal with sampling rate being M times that of the input signals. The next step is to transfer the signal through a channel. Finally, in the analysis bank the original signals are reconstructed with the aid of M analysis filters $H_k(\mathrm{z})$ for k = 0,1,…, M-1. These signals have the original sampling rates due to the decimation by a factor of M. If the output signal in the analysis-synthesis system is just a delayed version of the input signal, then for the corresponding transmultiplexer the output signals in the case of the ideal channel are delayed versions of the inputs. Therefore, the design of a transmultiplexer can be converted to the design of an analysis-synthesis filter bank.

The transmultiplexer system can be considered as a TDM (Time Division Multiplexing) to FDM (Frequency Division Multiplexing) converter, where several low rate signals are combined into a single high rate signal, for transmission purposes.

An analysis bank can be considered to perform a FDM to TDM conversion, in which a signal is divided into M subbands each having M-times lower sampling frequency compared to the input signal. Likewise, a synthesis bank performs a TDM to FDM conversion, in which M signals are modulated into a single signal having M-times higher sampling frequency than the input signals.

The M-channel transmultiplexer system consist of a synthesis filter bank followed by an analysis filter bank as shown in Fig. (50). The input M signals $X_k(z)$ for k = 0,1,……, M-1 are first interpolated by M, then filtered by the synthesis filters $F_k(z)$, and finally summed to form the channel signal $Y(z)$. At the receiver end, the channel signal is filtered by the analysis filters $H_k(z)$ and decimated by M in order to obtain the reconstructed output signals $Y_k(z)$. So, the same operations as in the analysis-synthesis configuration are used, but in the reverse order.

Here $F_k(z)$ are called transmitting filters or interpolation filters, and $H_k(z)$ are called the receiving filters or decimation filters.



Figure(50)
The transmultiplexer system.

The input output relation for the transmultiplexer system is given by

$$Y_k(z)=\frac{1}{M}\sum_{l=0}^{M-1}H_k(z^{\frac{1}{M}}W^l)\sum_{i=0}^{M-1}F_i(z^{\frac{1}{M}}W^l)X_i(z). \tag{5.1}$$

The analysis and synthesis filter banks shown in Fig. (50) can be redrawn again in terms of the polyphase representation $E(z)$ and $R(z)$. The resulting transmultiplexer structure is shown in Fig. (51) and an even more simplified structure in Fig. (52). As we can see, the filtering operations are again done at the lower sampling rate, i.e., at the subchannel sampling rates.

All examples in the previous chapter can be used as a transmultiplexer system, which can be used to convert TDM signal to FDM signal and then to TDM signal.

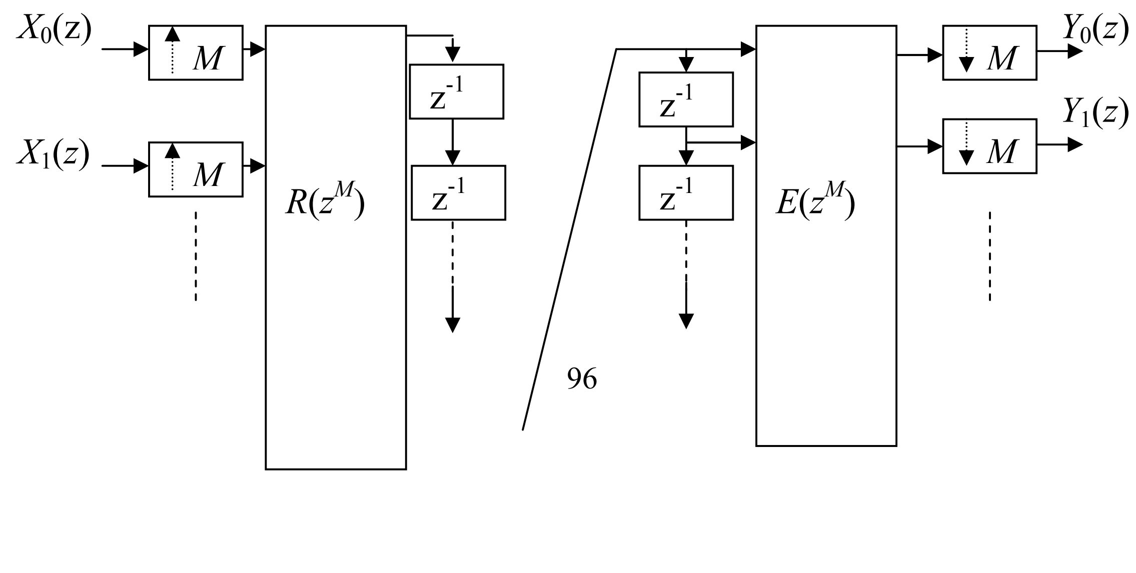




Figure (51)
Polyphase representation of an *M*-channel transmultiplexer system.

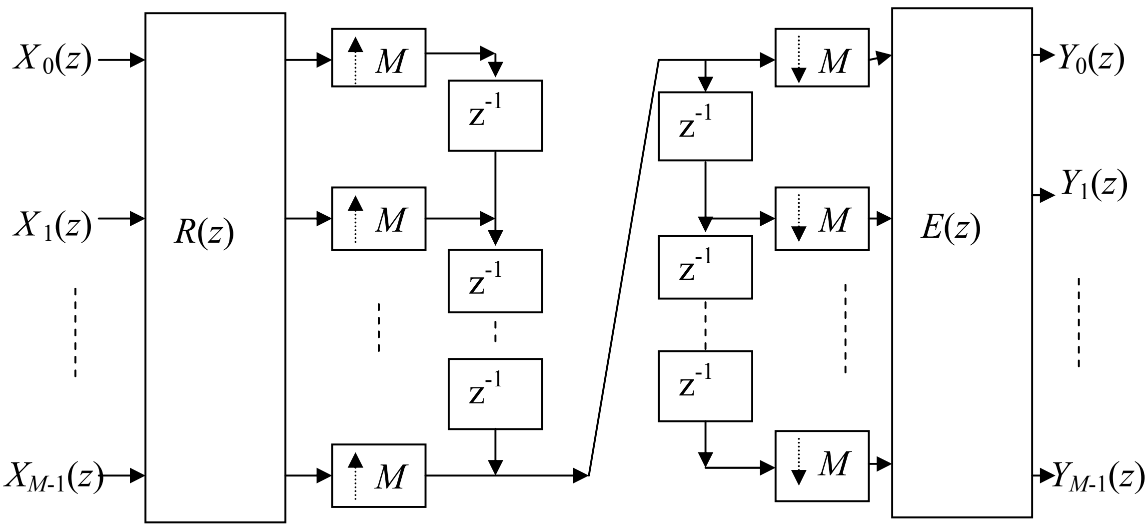


Figure (52)
Polyphase representation of an *M*-channel transmultiplexer system after some rearrangement by using noble identities.

5.3 Discrete Multitone Modulation (DMT)

The transmultiplexer configuration is used in another scheme called discrete multitone modulation or DMT scheme. The main difference is in the interpretation of the signals $x(n)$ and $x_k(n)$. To explain this the first stage of multitone modulation called the parsing stage. Here $s(n)$ represents binary data to be transmitted over a channel. This data is divided into nonoverlapping b-bit blocks. The $b$bits in each block are partitioned into M groups, the kth group being a collection of b_k bits.

The b_k bits in the kth group constitute the kth symbol x_k which can therefore be regarded as a b_k-bit number. For the nth block, this symbol is denoted as $x_k(n)$. This is the modulation symbol for the kth band. The collection of symbols $\{x_0(n), x_1(n), \dots ,x_{M-1}(n)\}$ is together referred to as the DMT symbol.

The sample $xk(n)$ is typically a PAM or a QAM symbol (Fig. 2). The transmitting filters $f_k(n)$ create the M-fold higher rate signals $u_k(n)$ as before, which are then added to produce the composite signal $x(n)$. In this way, various parts of the original binary message $s(n)$ are packed into different frequency regions allowedby the channel.

Notice that for a given constellation, the power can be increased or decreased by scaling the distance between the codewords. We therefore have the freedom to allocate different powers for different subband channels. In this way the classical water-filling rule can be approximated. For a given transmitted power and probability of error, multitone modulation yields better bit rate than single tone modulation (M = 1 case), assuming no channel coding.

5.4 Filter Bank Precoders

In a DMT system the symbols $xk(n)$ are obtained by parsing binary data $s(n)$. Instead of this, imagine that the symbols $x_k(n)$ are obtained by blocking a scalar signal $s(n)$ which itself belongs to a PAM or QAM constellation. In this system, a sequence of symbols $s(n)$ is converted to another sequence $x(n)$ before being fed into the channel.

If we assume that two successive samples of $x_k(n)$ are spaced apart by one second, then the samples of $s(n)$ are separated only by $1/M$ seconds and the channel input samples $x(n)$ are separated by the even smaller duration of $1/N$ seconds. This is a way of incorporating redundancy into a signal before putting it on the channel.

There are many applications which can be described with the help of the filter bank precoder configuration. If $s(n)$ comes from a finite field and all the arithmetic operations in the filtering are finite field operations,

then we can derive traditional channel coders such as block coders and convolutional coders as special cases of this system. These introduce redundancy in order to make the best use of the noisy channel. A more recent application is the use of such redundancy in channel equalization. The channel $C(z)$ can usually be approximated well by an FIR or stable IIR filter. The zero forcing equalizer $1/C(z)$ is in general IIR and could even be unstable (poles outside the unit circle). When we introduce redundancy as above, the use of an IIR .lter to approximate $1/C(z)$ is unnecessary. In a beautiful paper [37] Xia has shown that for almost any channel (FIR or IIR) there exist FIR filters $A_k(z)$ and $B_k(z)$ such that the channel is completely equalized (i.e., the received signal $r(n)$ is equal to $s(n)$ in absence of channel noise $e(n)$). In fact the well known class of fractionally spaced equalizers (FSE) is a special case of the filter bank precoder with $M = 1$ and uses N-fold redundancy. The fascinating fact about the filter bank precoder is that even if $M = N . 1$ it is almost always possible to have such FIR equalizers; by making N arbitrarily large we can therefore reduce the bandwidth expansion factor $N/(N . 1)$ to almost unity and still have FIR equalization.

All the above discussions assume that the channel transfer function $C(z)$ is known. There are many situations where this is not true. In these cases the removal of intersymbol interference from the received signal falls under the category of blind equalization. This redundancy introduced by filter bank precoders can actually be exploited to perform blind equalization. Further detailed results on blind as well as non-blind equalization with filter banks.

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